An investigation report on tensile and softening behavior of fiber reinforced concrete

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Abstract:

The project aims to study tensile softening behavior of fiber reinforced concrete .Concrete is very much weak in tension but by addition of randomly oriented short crimped steel fibers will change the behavior from brittle to ductile. In the present work ,the steel fibers were added at the volume fraction, being0%, 0.5%,1%,1.5% to the normal strength concrete and high strength concrete. The effects of steel fibbers on the tensile behaviour of high and normal strength are investigated. In the project work the tensile behaviour of concrete reinforced with steel fiber contents was assessed performing direct tensile tests .The fracture energy of conventional SFRC was independent of the specimen size. The fracture energy of SFRC with high strength matrix and normal strength matrix was dependent on the tensile strength of the steel fibers. From the results found that with an increase in % of fibers the tensile softening behavior increases and fracture energy also increases.

CHAPTER – 1

INTRODUCTION

Concrete, a composite consisting of aggregates enclosed in a matrix of cement paste including possible pozzolans, has two major components – cement paste and aggregates. The strength of concrete depends upon the strength of these components, their deformation properties, and the adhesion between the paste and aggregate surface . With most natural aggregates, it is possible to make concretes upto 120 MPa compressive strength by improving the strength of the cement paste, which can be controlled through the choice of water-content ratio and type and dosage of admixtures . However, with the recent advancement in concrete technology and the availability of various types of mineral and chemical admixtures, and special super plasticizer, concrete with a compressive strength of up to 100 MPa can now be produced commercially with an acceptable level of variability using ordinary aggregates . These developments have led to increased applications of high-strength concrete (HSC) all around the globe.

HSC offers many advantages over conventional concrete. The high compressive strength can be advantageously used in compression members like columns and piles. Higher compressive strength of concrete results reduction in column size and increases available floor space. HSC can also be effectively used in structures such as domes, folded plates, shells and arches where large in-plane compressive stresses exist. The relatively higher compressive strength per unit volume, per unit weight will also reduce the overall dead load on foundation of a structure with HSC. Also, the inherent techniques of producing HSC generate a dense microstructure making ingress of deleterious chemicals from the environment into the concrete core difficult, thus enhancing the long-term durability and performance of the structure. Since the introduction of concrete with a compressive strength of 62 MPa in columns, shear walls and transfer girders of the Water Tower Place in Chicago in 1975, many applications of HSC in projects, ranging from transmission poles to the tallest building (KLCC Twin Tower in Kuala Lumpur, Malaysia) on earth, with concrete strength reaching up to 131 MPa in the Union Square building in Seattle, Washington have been reported. ACI 363 (1992), CEB-FIP (1994), FIP/CEB (1990) and Russell (1994) have summarized worldwide development and use of HSC to demonstrate its versatility and wide ranging application potentials.

1.1:UNI AXIAL TENSILE TEST

The tension test is one of the most commonly used tests for evaluating materials. In its simplest form, the tension test is accomplished by gripping opposite ends of a test item within the load frame of a test machine. A tensile force is applied by the machine, resulting in the gradual elongation and eventual fracture of the test item. During this process, force-extension data, a quantitative measure of how the test item deforms under the applied tensile force, usually are monitored and recorded. When properly conducted, the tension test provides force-extension data that can quantify several important mechanical properties of a material.

These mechanical properties determined from tension tests include, but are not limited to, the following:

- Elastic deformation properties, such as the modulus of elasticity (Young‘s modulus)and Poisson‘s ratio
- Yield strength and ultimate tensile strength
- Ductility properties, such as elongation and reduction in area
- Strain-hardening characteristics

These material characteristics from tension tests are used for quality control in production, for ranking performance of structural materials, for evaluation of newly developed alloys, and for dealing with the static-strength requirement of design.

The basic principle of the tension test is quite simple, but numerous variables affect results. General sources of variation in mechanical-test results include several factors involving materials, namely, methodology, human factors, equipment, and ambient. methodology of the tension test and the effect of some of the variables on the tensile properties determined.

The following methodology and variables are discussed:

- Shape of the item being tested
- Method of gripping the item
- Method of applying the force
- Determination of strength properties other than the maximum force required to fracture the test item

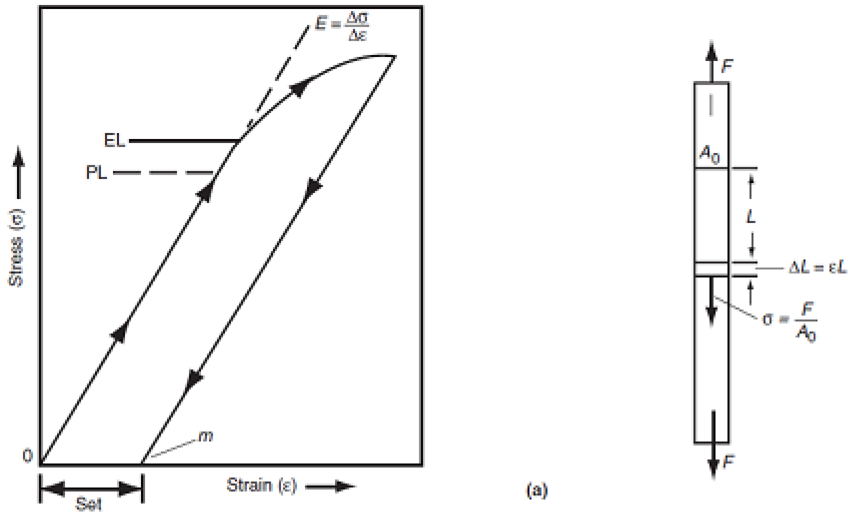

Fig 1.1:Stress-strain behavior in the region of the elastic limit. (a) Definition of σ and ε in terms of initial test piece length, L, and cross-sectional area, A0, before application of a tensile force, F. (b) Stress-strain curve for small strains near the elastic limit (EL)

1.2 Fibre reinforced concrete:

Fibre reinforced concrete (FRC) may be defined as a composite materials made with Portland cement, aggregate, and incorporating discrete discontinuous fibres.

Now, why would we wish to add such fibres to concrete? Plain, unreinforced concrete is a brittle material, with a low tensile strength and a low strain capcity. The role of randomly distributes discontinuous fibres is to bridge across the cracks that develop provides some post- cracking ―ductility‖. If the fibres are sufficiently strong, sufficiently bonded to material, and permit the FRC to carry significant stresses over a relatively large strain capacity in the post-cracking stage.

When the fibre reinforcement is in the form of short discrete fibres, they act effectively as rigid inclusions in the concrete matrix. Physically, they have thus the same order of magnitude as aggregate inclusions; steel fibre reinforcement cannot therefore be regarded as a direct replacement of longitudinal reinforcement in reinforced and prestressed structural members. However, because of the inherent material properties of fibre concrete, the presence of fibres in the body of the concrete or the provision of a tensile skin of fibre concrete can be expected to improve the resistance of conventionally reinforced structural members to cracking, deflection and other serviceability conditions.

The fibre reinforcement may be used in the form of three – dimensionally randomly distributed fibres throughout the structural member when the added advantages of the fibre to shear resistance and crack control can be further utilised . On the other hand, the fibre concrete may also be used as a tensile skin to cover the steel reinforcement when a more efficient two – dimensional orientation of the fibres could be obtained.

1.3:STATIC MECHANICAL PROPERTIES OF FIBERS

Compressive Strength

Fibres do little to enhance the static compressive strength of concrete, with increases in strength ranging from essentially nil to perhaps 25%. Even in members which contain conventional reinforcement in addition to the steel fibres, the fibres have little effect on compressive strength. However, the fibres do substantially increase the post-cracking ductility, or energy absorption of the material.

Tensile Strength

Fibres aligned in the direction of the tensile stress may bring about very large increases in direct tensile strength, as high as 133% for 5% of smooth, straight steel fibres. However, for more or less randomly distributed fibres, the increase in strength is much smaller, ranging from as little as no increase in some instances to perhaps 60%, with many investigations indicating intermediate values .Splitting-tension test of SFRC show similar result. Thus, adding fibres merely to increase the direct tensile strength is probably not worthwhile. However, as in compression, steel fibres do lead to major increases in the post-cracking behaviour or toughness of the composites.

Flexural Strength

Steel fibres are generally found to have aggregate much greater effect on the flexural strength of SFRC than on either the compressive or tensile strength, with increases of more than 100% having been reported. The increases in flexural strength is particularly sensitive, not only to the fibre volume, but also to the aspect ratio of the fibres, with higher aspect ratio leading to larger strength increases. the fibre effect in terms of the combined parameter Wl/d, where l/d is the aspect ratio and W is the weight percent of fibres. It should be noted that for $Wl/d > 600$, the mix characteristics tended to be quite unsatisfactory. Deformed fibres show the same types of increases at lower volumes, because of their improved bond characteristics.

1.4: STRUCTURAL USE AND BEHAVIOUR OF SFRC

As recommended by ACI Committee 544, _when used in structural applications, steel fibre reinforced concrete should only be used in a supplementary role to inhibit cracking, to improve resistance to impact or dynamic loading, and to resist material disintegration. In structural members where flexural or tensile loads will occur the reinforcing steel must be capable of supporting the total tensile load'. Thus, while there are a number of techniques for predicting the strength of beams reinforced only with steel fibres, there are no predictive equations for large SFRC beams, since these would be expected to contain conventional reinforcing bars as well. An extensive guide to design considerations for SFRC has recently been published by the American Concrete Institute. In this section, the use of SFRC will be discussed primarily in structural members which also contain conventional reinforcement.

For beams containing both fibres and continuous reinforcing bars, the situation is complex, since the fibres act in two ways:

(1) They permit the tensile strength of the SFRC to be used in design, because the matrix will no longer lose its load-carrying capacity at first crack; and

(2) They improve the bond between the matrix and the reinforcing bars by inhibiting the growth of cracks emanating form the deformations (lugs) on the bars

Flexure

The use of fibres in reinforced concrete flexure members increases ductility, tensile strength, moment capacity, and stiffness. The fibres improve crack control and preserve post cracking structural integrity of members.

Torsion

The use of fibres eliminate the sudden failure characteristic of plain concrete beams. It increases stiffness, torsional strength, ductility, rotational capacity, and the number of cracks with less crack width.

High Strength Concrete

Fibres increases the ductility of high strength concrete. Fibre addition will help in controlling cracks and deflections.

Shear

Addition of fibres increases shear capacity of reinforced concrete beams up to 100 percent. Addition of randomly distributed fibres increases shear-friction strength and ultimate strength.

Column

The increase of fibre content slightly increases the ductility of axially loaded specimen. The use of fibres helps in reducing the explosive type failure for columns.

Cracking and Deflection

Tests have shown that fibre reinforcement effectively controls cracking and deflection, in addition to strength improvement. In conventionally reinforced concrete beams, fibre addition increases stiffness, and reduces deflection.

1.5: Effects of fibres in concrete

Fibres are usually used in concrete to control cracking due to both plastic shrinkage and drying shrinkage. They also reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibres produced greater impact, abrasion and shatter resistance in concrete. Generally fibres do not increase the flexural strength of concrete and so cannot replace moment resisting or structural steel reinforcement. Indeed, some fibres actually reduce the strength of concrete. The amount of fibres added to the concrete mix is expressed as a percentage of total volume of the composite (concrete and fibres), termed volume fraction (Vf). Vf typically ranges from 0.1 to 3%. Aspect ratio (l/d) is calculated by dividing fiber length (l) by its diameter (d). fibres with a non circular cross section use an equivalent diameter for the calculation of aspect ratio. If the modulus of elasticity of the fibre is higher than the matrix (concrete or mortar binder), they help to carry the load by increasing the tensile strength of the material. Increase in the aspect ratio of the fibre usually segments the flexural strength and the toughness of the matrix. However, fibres which are too long tend to ―ball‖ in the mix and create workability problems. Some recent research indicated that using fibres in concrete has limited effect on the impact resistance of the materials. This finding is very important since traditionally, people think that the ductility increases when concrete is reinforced with fibres. The results also indicated out that the use of micro fibres offers better impact resistance compared with the longer fibres.

1.6:EFFECT ON WORKABILITY OF STEEL FIBER:

Slump tests were carried out to determine the workability and consistency of fresh concrete. The efficiency of all fiber reinforcement is dependent upon achievement of a uniform distribution of the fibers in the concrete, their interaction with the cement matrix, and the ability of the concrete to be successfully cast or sprayed. Essentially, each individual fiber needs to be coated with cement paste to provide any benefit in the concrete. Regular users of fiber reinforcement concrete will fully appreciate that adding more fibers into the concrete, particularly of a very small diameter, results in a greater negative effect on workability and the necessity for mix design changes. The slump changed due to the different type of fiber content and form. The reason of lower slump is that adding steel fibers can form a network structure in concrete, which restrain mixture from segregation and flow. Due to the high content and large surface area of fibers, fibers are sure to absorb more cement paste to wrap around and the increase of the viscosity of mixture makes the slump loss.

1.7:ADVANTAGES AN APPLICATIONS OF FIBERS

Advantages

- High modulus of elasticity for effective long-term reinforcement, even in the hardened concrete.
- Does not rust nor corrode and requires no minimum cover.
- Ideal aspect ratio (i.e. relationship between Fiber diameter and length) which makes them excellent for early-age performance.
- Easily placed, Cast, Sprayed and less labour intensive than placing rebar.
- Greater retained toughness in conventional concrete mixes.
- Higher flexural strength, depending on addition rate.
- Can be made into thin sheets or irregular shapes.
- FRC possesses enough plasticity to go under large deformation once the peak load has been reached**.**

The main area of FRC applications are:

a) Runway, Aircraft Parking, and Pavements

For the same wheel load FRC slabs could be about one half the thickness of plain concrete slab. Compared to a 375mm thickness' of conventionally reinforced concrete slab, a 150mm thick crimped-end FRC slab was used to overlay an existing as phaltic-paved aircraft parking area. FRC pavements are now in service in severe and mild environments.

b) Tunnel Lining and Slope Stabilization

Steel fiber reinforced shortcrete (SFRS) are being used to line underground openings and rock slope stabilization. It eliminates the need for mesh reinforcement and scaffolding.

c) Blast Resistant Structures

When plain concrete slabs are reinforced conventionally, tests shows that there is no reduction of fragment velocities or number of fragments under blast and shock waves. Similarly, reinforced slabs of fibrous concrete, however, showed 20 percent reduction in velocities, and over 80 percent in fragmentations.

d) Thin Shell, Walls, Pipes, and Manholes

Fibrous concrete permits the use of thinner flat and curved structural elements. Steel fibrous shortcrete is used in the construction of hemispherical domes using the inflated membrane process. Glass fiber reinforced cement or concrete (GFRC) , made by the spray-up process, have been used to construct wall panels. Steel and glass fibers addition in concrete pipes and manholes improves strength, reduces thickness, and diminishes handling damages.

e) Dams and Hydraulic Structure

FRC is being used for the construction and repair of dams and other hydraulic structures to provide resistance to cavitation and severe erosion caused by the impact of large water bore debris.

Other Applications are include machine tool frames, lighting poles, water and oil tanks and concrete Repairs.

1.8: FRACTURE MECHANICS

Fracture mechanics is the field of mechanics concerned with the study of the propagation of cracks in materials. It uses methods of analytical solid mechanics to calculate the driving force on a crack and those of experimental solid mechanics to characterize the material's resistance to fracture.

Fracture mechanics plays a central role, as it provides useful tools which allow for an analysis of materials which exhibit cracks. The goal is to predict whether and in which manner failure might occur. Fracture mechanics is generally applied in the field of earth sciences such as petroleum engineering, geological engineering, mining engineering and civil engineering Materials like ceramics, rocks, glasses and concretes behave as brittle and in brittle materials; the crack initiation is determined by using the linear elastic stress field around the crack tip. This application belongs to Linear Elastic Fracture Mechanics (LEFM) assumption. This assumption is valid, when plastic deformation around the crack is negligible. Concrete is a stone like material obtained by permitting a mixture of cement, sand and gravel or other aggregate and water, to harden in forms of the desired shape of the structure. Concrete has become a popular material in civil engineering for several reasons, such as the low cost of the aggregate, the accessibility of the needed materials and its high compressive strength. On the other hand concrete is a relatively brittle material with low tensile strength compared to the compressive strength.

THE FRACTURE PROCESS IN TENSION

The tensile strength of concrete is much like the compressive strength, dependent on the strength of each link in the cracking process, i.e., micro cracks in the cement paste, micro cracks in the bond and macro cracks in the mortar. Consider a concrete rod under pure tensile loading. The fracture process initiates with crack growth of existing micro cracks at approximately 80% of the ultimate tensile load. This is followed by formation of new cracks and a halt in formation of others due to stress redistribution and the presence of aggregates in the crack path. These cracks are uniformly distributed throughout the concrete specimen. When the ultimate tensile load is reaches, a localized fracture zone will form in which a macro crack that splits the specimen in two will form. The fracture zone develops in the weakest part of the specimen.

According to ACI committee 446-*Fracture mechanics of concrete*, there are five reasons that fracture mechanics can improve some aspects of structural concrete design (ACI 446 1991).

Crack Formation

Fracture mechanics is an energy based method, in which the formation of cracks requires that the material must absorb some amount energy (Bazant and Planas 1998) associated with the resistance of that material (Anderson 2005). This energy requirement is important, because it implies that if it is not met the material will not fracture even after the design strength is met (Bazant and Planas 1998).

a) **Analysis must be Objective**:

The example provided by ACI 446 is a problem that occurs in finite element models of concrete fracture, known as spurious mesh sensitivity. It arises when using the smeared crack approach, in which the results of the model and method of analysis depends primarily on the mesh selection (ACI 446 1991).

b) **Absence of Yield Plateau**:

In some instances the use of a plastic limit analysis, as is often done, cannot account for the existence of softening or hardening behavior in the absence of a yield plateau (Bazant and Planes 1998).

c) **Fracture Toughness**:

The fracture toughness of the structure is approximated by the area under the load displacement curve. As mentioned, plastic limit analysis does not account for any softening behavior in the material due to the energy dissipated during fracture. This softening behavior leads to a finite amount of fracture toughness in the structure, whereas the fracture toughness using plastic limit analysis is, in theory, unlimited (Bazant and Planas 1998).

d) **Size Effect**:

As will be discussed, concrete falls victim to the size effect, which is a phenomenon that occurs when comparing strengths of structures that exhibit geometric similitude. It follows that as the size of the structure increases the strength and ductility decrease. This effect is not accounted for in plastic limit analysis, but could be with the inclusion of fracture mechanics (Bazant and Planas1998).

1.8.1: FRACTURE BEHAVIOR:

The quasi-brittle behavior of concrete can be best explained by the following five stages (Shah et al. 1995) as depicted graphically in figure 1.9.1 and with the use of figure1.9.2.

Elastic:

The material exhibits elastic behavior until the proportional elastic limit (PEL) is reached. The PEL in concrete is typically assumed to be the point of first crack (Shah et al. 1995).

Micro Cracking:

Random micro – cracking occurs ahead of a flaw leading to a toughening behavior (Shah et al. 1995).

Damage Localization:

The micro-cracks will localize forming a micro-crack, which occurs at the point of initial crack localization. At which point the material undergoes stable crack growth (crack propagates only when load increases) and a softening behavior occurs (Shah et al. 1995).

Unstable Crack Growth:

Once the ultimate strength is reached at a critical crack length the crack will undergo unstable growth (crack propagates even though load decreases) (Shah et al. 1995).

Failure:

The crack will continue to propagate until failure, which occurs when the stress is equal to zero (Shah et al. 1995).

The region ahead of the initial flaw location is termed the fracture process zone (Shah et al. 1995). This zone can be separated into the crack wake process zone and the crack tip process zone. As shown in Figure 3b, the crack tip process zone is the region ahead of the crack where micro-cracking occurs during the fracture process and the crack wake process zone is the region in which the macro-crack resides (Shah et al.1995).

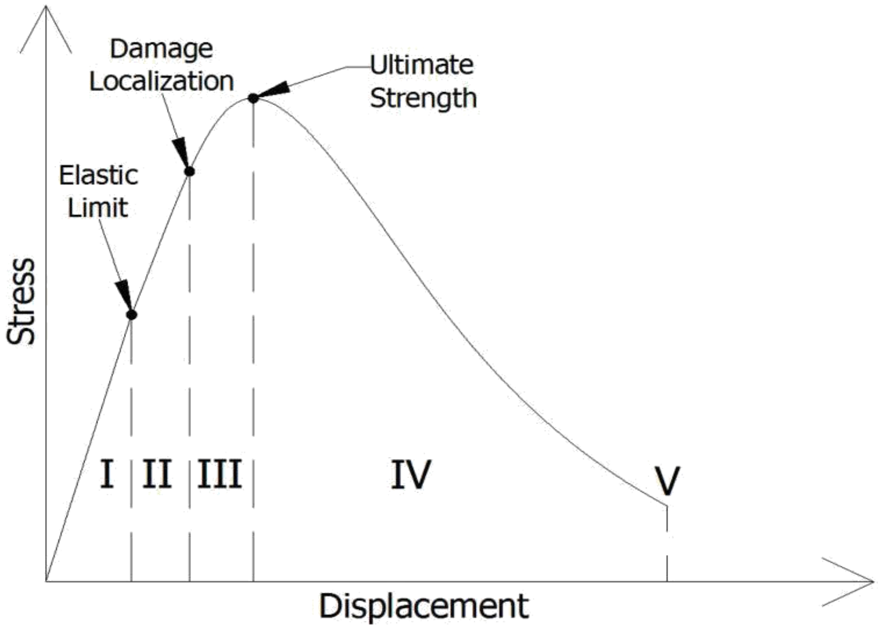

Fig. 1.8.1 Stages of quasi-brittle behaviour

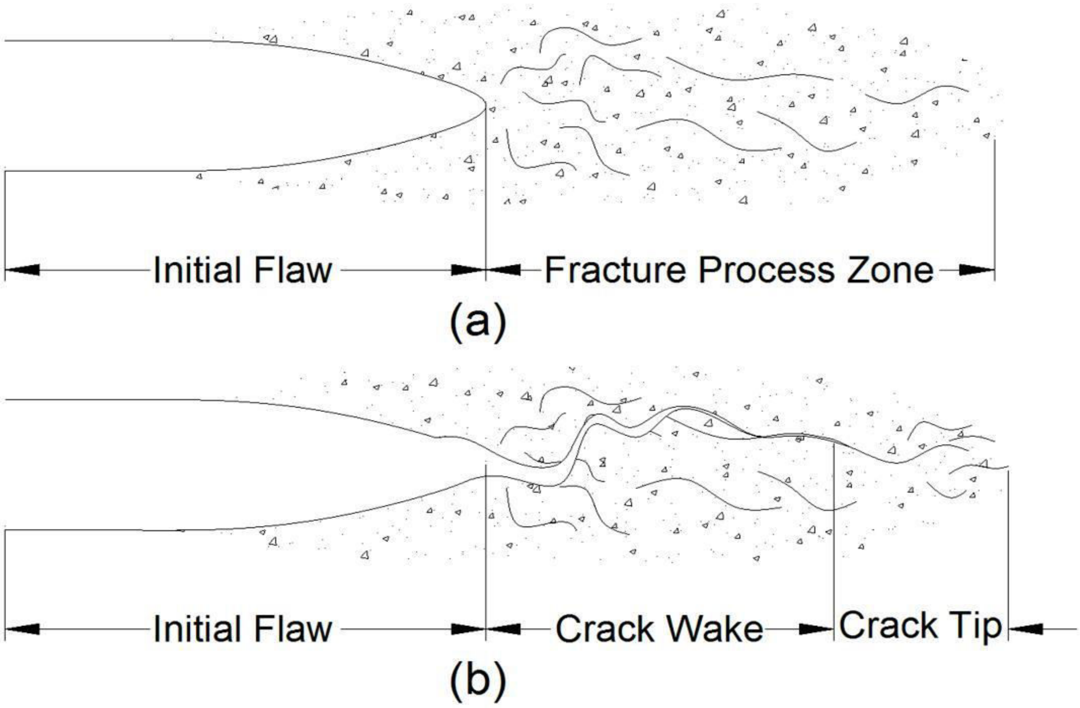

Fig1.8.2 Fracture process zone: stage II (a) and stage III (b)

For analysis purposes it is commonly assumed that a crack propagates in a linear fashion. However, concrete is a composite material so cracks tend to propagate along non-linear or chaotic crack paths due to the heterogeneity of the material. This can be associated with several toughening mechanisms that occur within the Fracture Process Zone (FPZ) as pointed out by Shah et al. (1995).

Micro crack Shielding:

Randomly oriented micro cracks occur at flaws ahead of the crack tip. The micro-cracking is caused by the high stress concentration near the crack tip (Shah et al. 1995). The formation of micro-cracks releases energy, which increases the amount of energy required to form unstable cracks (Anderson 2005).

Crack Deflection:

This occurs when inclusion (i.e., aggregates or fibers) is strong enough to divert the path of least resistance around the inclusion (Shah et al.1995).

Crack Bridging:

If an inclusion is bonded to the concrete at both cracks faces the inclusion has the ability to transfer stress across cracks (Shah et al. 1995). It has been stated that fiber bridging is the most effective toughening mechanism for brittle materials (Anderson 2005).

Surface Friction:

Surface interlock can cause energy dissipation due to friction between fracture surfaces (Shah et al. 1995).

Crack Tip Blunting:

This commonly occurs when a crack reaches a void (Shah et al. 1995). This is a form of crack arrest which occurs when the energy required to produce a crack is insufficient to overcome the materials resistance to fracture (Broek 1986).

Crack Branching:

Crack branching in concrete occurs due to heterogeneity of the material (Shah et al.1995). In an ideal situation the bifurcation of crack will occur, theoretically, when the fracture energy is twice that of the energy to resist fracture (Broek 1986). These toughening mechanisms are all sources of variability in the fracture behavior of concrete, and may explain the source of size effect (Shah et al. 1995).

1.8.3:MODES OF FRACTURE FAILURE:

A crack front in a structural component is a line usually of varying curvature. Thus, the state of stress in the vicinity of the crack front varies from one point to another. A segment of the crack front can be divided into 3 basic modes as shown in fig1.9.3.

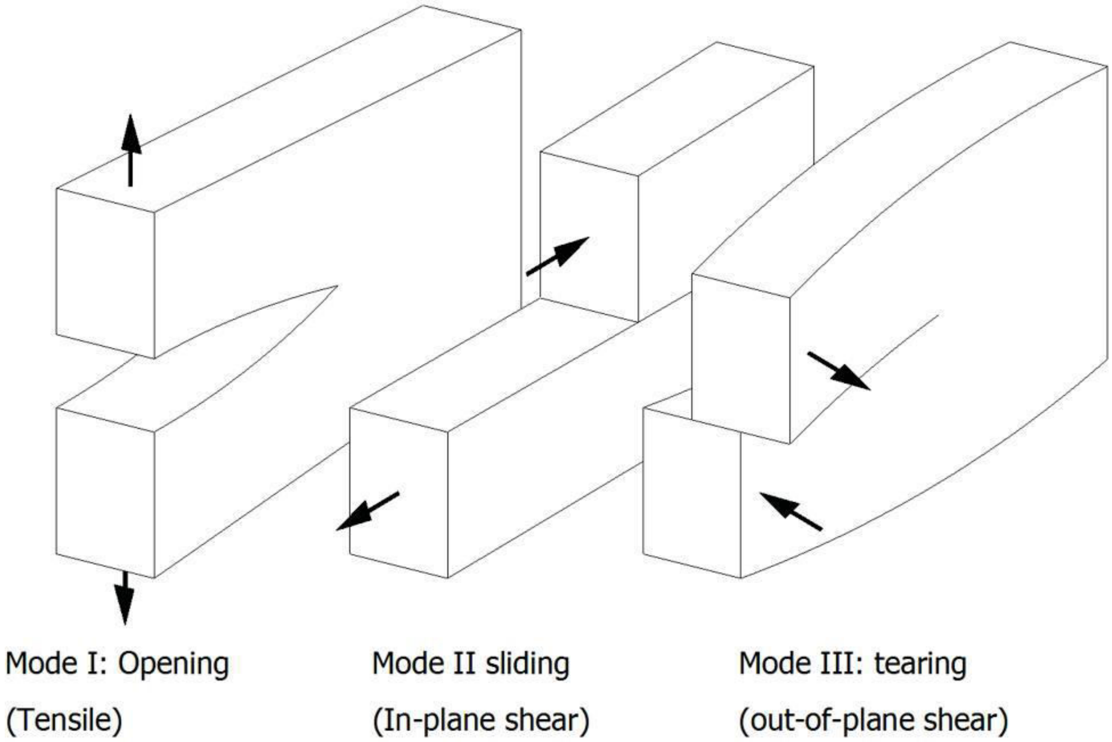

Fig1.8.3 the three modes of fracture

- **MODE I** – it is the opening mode and the displacement is normal to the crack surface.
- **MODE II** – it is the sliding mode and the displacement is in plane of the plate (the separation is antisymmetric and the relative displacement is normal to the crack front).
- **MODE III** – it is the tearing mode and the displacement is parallel to the crack front

CHAPTER – 2

LITERATURE

V. Mechtcherine and H.S. Miiller(1976),did an investigation the effect of the test set-up on fracture mechanical parameters of concrete was studied experimentally and numerically. In the first step a series of deformation controlled uniaxial tension tests on dog-bone shaped specimens and notched specimens. with rotatable and non-rotatable boundaries as well as three-point bend tests were performed. As a result, the experiments with rotatable loading platens provided lower values of the fracture energy Gp than the tests with nonrotatable boundaries, but slightly higher GF-values than those obtained from the bend tests. In the second step, these experiments were analyzed numerically within the frame of a smeared crack concept. The analysis showed, that the GF-values obtained from the uniaxial tension tests with non-rotatable loading platens are the most realistic ones.

The results summarized above allow to draw the following conclusions with regard to the determination of the input parameters for numerical analyses using cohesive crack type models:

- The tensile strength ft and the modulus of elasticity for the material law should be determined from uniaxial tension tests on unnotched specimens with non-rotatable boundaries.
- In ideal case, also the fracture energy GF and the shape of the softening curve should be obtained from such tests. However, it is much easier to perform stable experiments on notched specimens. Hereby, uniaxial tension tests with non-rotatable boundaries provide a
- descending branch of the cr-8-relation, which shows the best correspondence with the real softening behaviour of concrete.
- When deriving a stress-crack opening relation from the cr-8-relations obtained in tests with non-rotatable boundaries its first, steeper part should be corrected upwards to mach the tensile strength ft. As a result, also the value of the input fracture energy increases up to the
- correct value. The second, shallow part of the softening branch can be taken over directly from the measured cr-8-relations.

Milind V. Mohod(2012) studied on **Performance of Steel Fiber Reinforced Concrete.** In this modern age, civil engineering constructions have their own structural and durability requirements, every structure has its own intended purpose and hence to meet this purpose, modification in traditional cement concrete has become mandatory. It has been found that different type of fibers added in specific percentage to concrete improves the mechanical properties, durability and serviceability of the structure. It is now established that one of the important properties of Steel Fiber Reinforced Concrete (SFRC) is its superior resistance to cracking and crack propagation. In this paper effect of fibers on the strength of concrete for M 30 grade have been studied by varying the percentage of fibers in concrete. Fiber content were varied by 0.25%, 0.50%, 0.75%, 1%, 1.5% and 2% by volume of cement. Cubes of size 150mmX150mmX150mm to check the compressive strength and beams of size 500mmX100mmX100mm for checking flexural strength were casted. All the specimens were cured for the period of 3, 7 and 28 days before crushing. The results of fiber reinforced concrete for 3days, 7days and 28days curing with varied percentage of fiber were studied and it has been found that there is significant strength improvement in steel fiber reinforced concrete.

Following conclusions were drawn from the work carried out;

1. It is observed that the workability of steel fibre reinforced concrete gets reduced as the percentage of steel fibres increases.
2. Compressive strength goes on increasing by increase in steel fibre percentage up to the optimum value. The optimum value of fibre content of steel fibre reinforced concrete was found to be 1%.
3. The flexural strength of concrete goes on increasing with the increase in fibre content up to the optimum value. The optimum value for flexural strength of steel fibre reinforced cement concrete was found to be 0.75%.
4. While testing the specimens, the plain cement concrete specimens have shown a typical crack propagation pattern which leaded into splitting of beam in two piece geometry. But due to addition of steel fibres in concrete cracks gets ceased which results into the ductile behaviour of SFRC

Mazen Musmar studided on **Tensile Strength of Steel Fibber.**

Studies have shown that the addition of steel fibers in a concrete matrix improves all the mechanical properties of concrete, especially tensile strength, impact strength, and toughness.

The resulting material possesses higher tensile strength, consolidated response and better ductility.

The following conclusions can be drawn from this study: 1- A mathematical expression that predicts the split tensile strength of steel fibber reinforced concrete is derived.

2- The suggested equation correlates the split tensile strength of steel fibber reinforced concrete with concrete compressive strength and fiber reinforcement index.
3- The predicted values of the splitting tensile strength are in good agreement with the experimental results. Thus the validity of the suggested expression is verified against the experimental results gathered from previous researches.
4- The outcomes of descriptive statistical analysis confirm the credibility of the derived expression.
5- Concrete compressive strength, fiber content and the fiber aspect ratio are the major effectual parameters in specifying the tensile strength of fiber concrete.

Jordon R. Deluce and Frank J. Vecchio on **Cracking Behavior of Steel Fiber-Reinforced Concrete Members(2013)**

The observations made through this experimental program have led to the following conclusions:

1. Steel fibers added to concrete reinforced with conventional reinforcing bars improve the cracking characteristics and tension-stiffening behavior of the material compared to non fibrous RC.
2. Steel fibers can increase the post-yield load-carrying capacity of a uniaxial concrete tension member reinforced with conventional reinforcement to levels significantly higher than the bare-bar yield load.
3. An increase in fiber content tends to decrease the mean crack spacings and maximum crack widths of SFRC reinforced with conventional steel reinforcing bars
4. An increase in fiber aspect ratio tends to decrease the mean crack spacings and maximum crack widths of SFRC reinforced with conventional steel reinforcing bars.
5. An increase in reinforcement ratio of the conventional steel reinforcing bar decreases the mean crack spacings and maximum crack widths of both SFRC and nonfibrous concrete.

6. For a given reinforcement ratio, an increase in the conventional reinforcing bar diameter increases the mean crack spacings and maximum crack widths of both SFRC and non fibrous concrete.
7. Fiber length does not appear to play a significant role in the post-cracking behavior of SFRC containing conventional reinforcing bars, provided that the crack spacing is not so short that a fiber bridges multiple cracks.
8. The currently available crack spacing models are not adequate for calculating the mean crack spacing of R/SFRC members. Improved formulations are required.

Research being conducted in the by **FAISAL FOUAD WAFA(1990)**
Based on the test of one hundred and ninety five specimens

1. No workability problem was encountered for the use of hooked fibers up to 1.5 percent in the concrete mix. The straight fibers produce balling at high fiber content and require special handling procedure.
2. Use of fiber produces more closely spaced cracks and reduces crack width. Fibers bridge cracks to resist deformation.
3. Fiber addition improves ductility of concrete and its post-cracking load-carrying capacity.
4. The mechanical properties of FRC are much improved by the use of hooked fibers than straight fibers, the optimum volume content being 1.5 percent. While fibers addition does not increase the compressive strength, the use of 1.5 percent fibber increase the flexure strength by 67 percent, the splitting tensile strength by 57 percent, and the impact strength 25 times.
5. The toughness index of FRC is increased up to 20 folds (for 1.5 percent hooked fiber content) indicating excellent energy absorbing capacity.
6. FRC controls cracking and deformation under impact load much better than plain concrete and increased the impact strength 25 times

Dario Redaelli studied on **TESTING OF REINFORCED HIGH PERFORMANCE FIBRE CONCRETE MEMBERS IN TENSION(2006)**

A test series has been carried out on UHPFC reinforced with two types of ordinary steel reinforcement.

At the serviceability limit state, the behaviour of tensile members is very positively affected by the interaction between UHPFC and steel. Up to significant load levels, cracks are very thin and closely spaced. Moreover, tension stiffening is much more effective than in ordinary concrete, leading to a substantial increase in stiffness.

UHPFC tensile members cannot be made fully ductile by adding ordinary reinforcement. This results from the inherent mechanical properties of the materials. Alternative solutions need to be found, and three approaches have been proposed for future work.

ALI AMIN, STEPHEN J. FOSTER, AND AURELIO MUTTON studied on EVALUATION OF THE TENSILE STRENGTH OF SFRC(2013)

For the design of SFRC members, the most fundamental material property is its post cracking residual tensile strength. When relying on physical models to describe structural behaviour under load, the material laws must first be accurately established. If the material laws either significantly over- or under-estimate the residual tensile capacity of the SFRC, an accurate physical model for the determination of its strength is not possible. However, in production control and materials specification, direct tensile testing is costly in time and difficult in that it requires specialised testing equipment. To this end, testing of prisms in bending is often substituted for uniaxial tension testing, and empirical design models are developed based on the results. Several attempts have been undertaken to provide an inverse analysis from prism data to establish the σ-w relationship but no direct test verification of the approaches have, to date, been established. This paper compares the results for σ-w relationship obtained from a direct tension test with those obtained using prism tests combined with an inverse analysis. Finally, a model is proposed that can be used for control testing with designs established using physically based models for strength limit states.

K.S. Prebhakumari, P. Jayakumar(2013) This experimental investigation on the fracture behaviour of high strength concrete and steel fibre reinforced high strength concrete with particular emphasis on the size effect method. Fracture study was carried out by conducting three point bending tests on series of geometrically similar single edge notched beams. The influence of notch size on the fracture properties of steel fibre reinforced high strength concrete was also investigated. Various fracture parameters like the fracture energy, length of fracture process zone, critical crack tip opening displacement and the fracture toughness were determined as per RILEM procedure. The test results showed that the fracture parameters are sensitive to the fibre addition and the notch size. With the experimental parameters an attempt has been made to predict the nominal strength of steel fibre reinforced high strength concrete structures.

***Serdar* Aydın** studied **on Effects of fiber strength on fracture characteristics of normal and high strength concrete(2012)**

Based on the experimental results of this investigation the following conclusions can be drawn:

- The mechanical mismatch between steel fibers and concrete have a significant role in fracture behavior of steel fiber reinforced concretes. For high strength concrete, usage of high
- Strength steel fiber with a tensile strength of 2000MPa recommended according to the test results. However, high strength steel fiber usage seems to be unnecessary for normal strength concrete.
- Splitting tensile strength, flexural strength, fracture energy and toughness indexes of the steel fiber reinforced high strength concrete have been significantly improved by fibber strength. The improvement of mechanical properties and fracture behavior of high strength concrete by using high strength fibers is related to the lesser number of broken fibers and increased debonding process.
- As a general conclusion, high strength steel fibers can be used preferably in high strength fiber reinforced concrete in two ways. If high strength steel fibers are added with the same dosage of normal strength fiber, flexural performance of composites improves significantly; in this case they act as mechanical performance developer. The other way is the
- Reduction of steel fiber dosage compared to normal strength fibers. In this case, similar mechanical performance with normal strength fiber can be obtained by inclusion of less amount of fibers. This provides the production of a more economical fiber reinforced concrete due to the reduction of fiber dosage.
- Besides, it improves the workability of concrete at a constant superplasticizer dosage or it can reduce the superplasticizer dosage in case of constant workability

CHAPTER – 3

EXPERIMENTAL PROGRAM

The experimental program is designed to understand whether the addition of fibers in high strength concrete and normal strength concrete favours strain hardening and increase of amount of fibers produces identical enhancement of tensile properties.

3.1:Materials :

The main ingredients used were cement, fine aggregate, coarse aggregate, water, super plasticizer and steel fibres.

Cement: Ordinary Portland Cement of 53 grade conforming to IS: 12269-1987 was used for the study. The cement content can be 350 – 450 kg/m3. Some amount of cement replaced by adding add-mixtures to increase strength and durability

Water: Potable water supplied by the college was used in the work

Fine Aggregate: River sand passing through 4.75 mm sieve and conforming to grading zone II of IS: 383-1970 was used as the fine aggregate. Normal river sands are suitable for high strength concrete. Both crushed and rounded sands can be used. Siliceous and calcareous sands can be used for production of HSC

Coarse Aggregate: Crushed granite stone with a maximum size of 20 mm was used as the coarse aggregate. The properties of aggregates used

Super Plasticizer: Conplast SP430 a product of Fosroc was used as the super plasticizer.

Steel Fibre: Crimped steel fibres with 0.35 mean diameter was used at a volume fraction of 0%, 0.5%,1%,1.5%

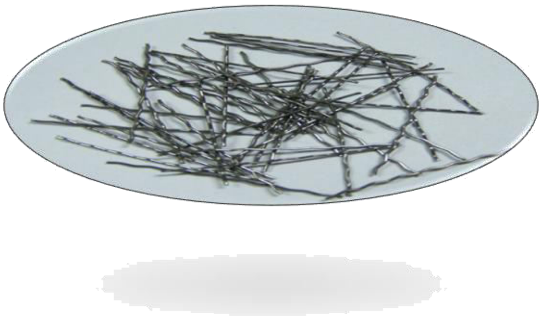

FIG 3.1.1 steel crimped fibers

Tamping Rod : Tamping rod was used for compacting the test specimens, beams.

Mould shape:

Fig 3.1.2 : specimen mould shape.

Casting: The moulds were tightly fitted and all the joints were sealed by bolts and nut order to prevent leakage of cement slurry through the joints. The inner side of the moulds was thoroughly oiled . The mix proportions were put in miller and thorough While casting the specimen , most of the concrete required was poured in the middle of the mould and allowed to spread to the ends; few scoops of concrete were placed at the ends to top of the mould. This method was followed so as to avoid any weak planes in the zone where failure is expected to occur during testing.

Fig 3.1.3: casted specimen

Curing: The specimens were removed from the moulds after 24 hours of casting and the specimens were placed in water for curing

Preparing of Notch: The notch was prepared with steel plates with different a/w ratio sizes.

3.2: MIX PROPORTIONING

The normal strength concrete mix M30 was proportioned as per Indian Standard for a target mean strength 30MPa. After various trial mixes, the optimum mix proportion was selected as 0.45:1:1.562:2.902 with cement content of 405.81 kg/m^3. The different constituents in the order of water: cement: fine aggregate: coarse aggregate were proportioned as 60.32:134.11:209.53:389.11 for making 1m^3 of mix.

The high strength concrete mix M70 was proportioned as per indian standards for target mean strength 70MPa. After various trial mixes, the optimum mix proportion was selected as 0.24:1:1.346:1.103 with cement content of 650.58kg/m^3. The different constituents in the order of water: cement: fine aggregate: coarse aggregate were proportioned as 60.32:134.11:209.53:389.11 for making 1m^3 of mix.

3.3:Specimen preparation:

A operated miller of sixty litre capacity was used to prepare the cement mixture. Cement, water, coarse aggregate, sand ,ground granulated blast furnace slag, crimped fibre and super plasticizer were used. Cement sand silica fume, GGBS were first dry mixed for about 10 min. water pre mixed with conplast sp430 was added gradually and mixed for another 5-10 min

.when the mortar show enough flow ability for workability and viscosity for uniform fiber distribution , the crimped fiber were dispersed carefully by hand into the mortar mixture added .the cement mixture with fibers was then placed in mould .the specimen s were placed in the water curing for 2 days after remoulding was carried out .all specimens were tested in dry condition for 28 days

Table 1: Details of materials for 1 cubic meter of concrete for M30

Grade of concrete	Mix Proportion	Water wt. (kg)	Cement wt.(kg)	Weight of FA (kg)	Weight of CA (kg)
M30	0.45:1:1.562:2.902	183.0	406.81	635.4	1180.56

Table 2: Details of materials for 1 cubic meter of concreteM70

Grade of concrete	Mix Proportion	Water wt. (kg)	Cement wt.(kg)	Weight of FA (kg)	Weight of CA (kg)
M70	0.24:1:1.36:1.103	156.13	650.58	875.68	717.5

Table 3:details of materials for M70

Grade of concrete	% of fibres	water	cement	Fine aggregate	Coarse aggregate	Ggbs (kg)	Micro silica (kg)	Super plastisizer (ml)	% of fibers
M70	0								0.000
	0.5								0.344
	1.0	2.73	11.41	15.36	12.58	0.798	1.48	21.84	0.688
	1.5								1.033

Table 4 : details of materials for M30

Grade of Concrete	% of fibres	water	Cement	Fine aggregate	Coarse aggregate	% of fibers
	0					0.000
	0.5					0.344
M30	1.0	3.204	7.12	11.12	20.66	0.688
	1.5					1.033

Chapter 4

TEST SETUP AND TESTING PROCEDURE

4.1 Test setup and specimen

The dimensions of the test specimen were chosen to be representative of actual structural Element and to provide a cross section large enough to place various types and amounts of Fiber .The load introduction system was designed to prevent the development of eccentricities or unexpected end rotations. Rigid end conditions were chosen as the best solution from a constructive point of view.

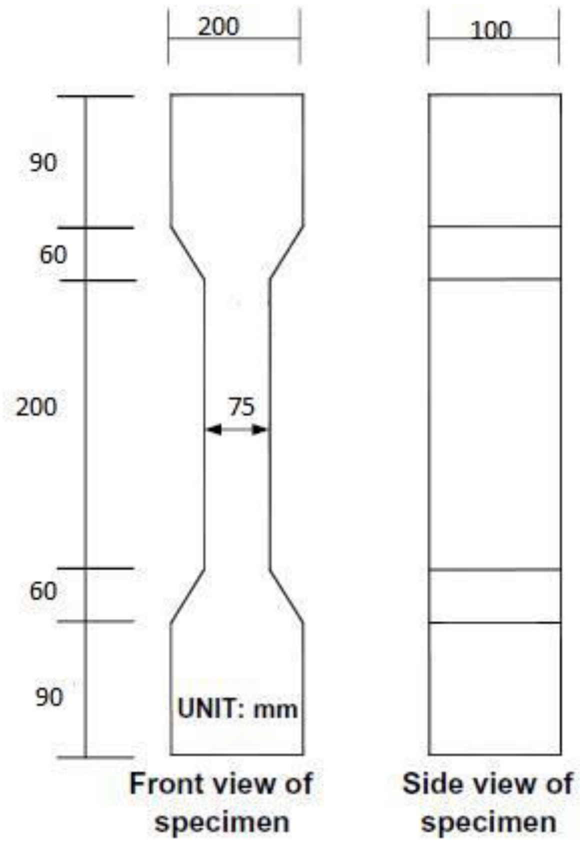

Fig:4.1: dimensions of specimen

4.2 Tensile Test:

To obtain increased understanding about normal strength steel fiber reinforced concrete and high strength steel fiber reinforced concrete behavior program, under direct-tensile load, a conventional dogbone shape specimen was selected to detect elongation occurring during the test. The notch was provided at the center .At the centre at equal distance dail gauge was fixed to detect the elongation The top and bottom ends of the specimens were held by specially designed grips attachedof rod size 25 mm diametre was fixed inoder to fit into the machine The average elongation was obtained from dial guage.this process was carried on universal testing machine.

4.2: Test set up of specimen

4.2.1 :specimen testing on utm

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CHAPTER – 5

RESULTS AND DISCUSSIONS

The specimens were tested on the Universal Testing Machine under deflection rate control. All the specimens were tested under the uniaxial tension test under deflection rate control. To understand the fracture behaviour of plain and fiber reinforced concrete specimen the following graphs were drawn, Load Vs deflection. The stress strain and fracture energy of subjected to tensile test. calculated by using the graphs and Tables it was observed that, for tensile failure of concrete, It was found that the stress intensity factor and fracture energy increases with the increasing of % of fibers.

Based on the tests on specimen can be observed that, in the case of centred notched specimen , the first crack appeared in the tension zone at notch tip. The deflection were measured only up to the ultimate load and failed suddenly in to two pieces

The tensile stress-strain curves (where the strain is valid up to the peak stress only) for the test series and are given in the graph and discussed below. The load increased with increased displacement

GRADE OF CONCRETE	% OF FIBRES	POST PEAK AREA
M30	0	0.076
	0.5	0.144
	1	0.455
	1.5	0.828
M70	0	0.274
	0.5	0.502
	1	0.668
	1.5	0.6747
	1.5	0.935

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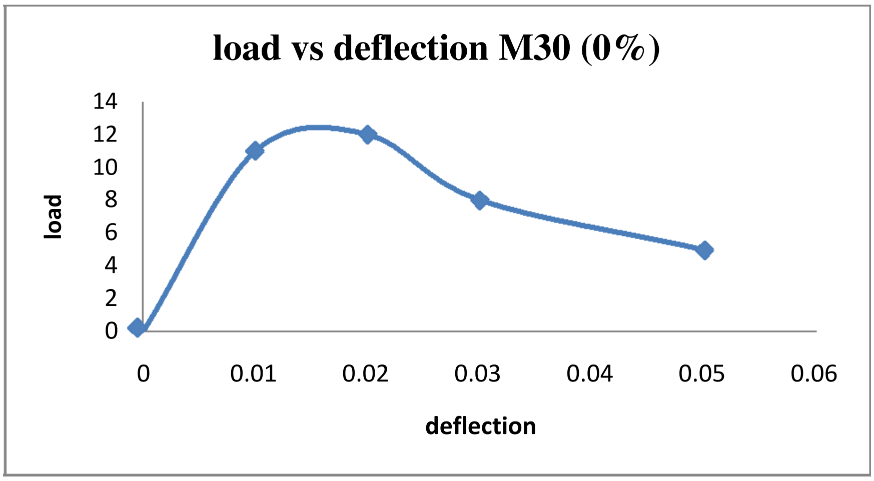
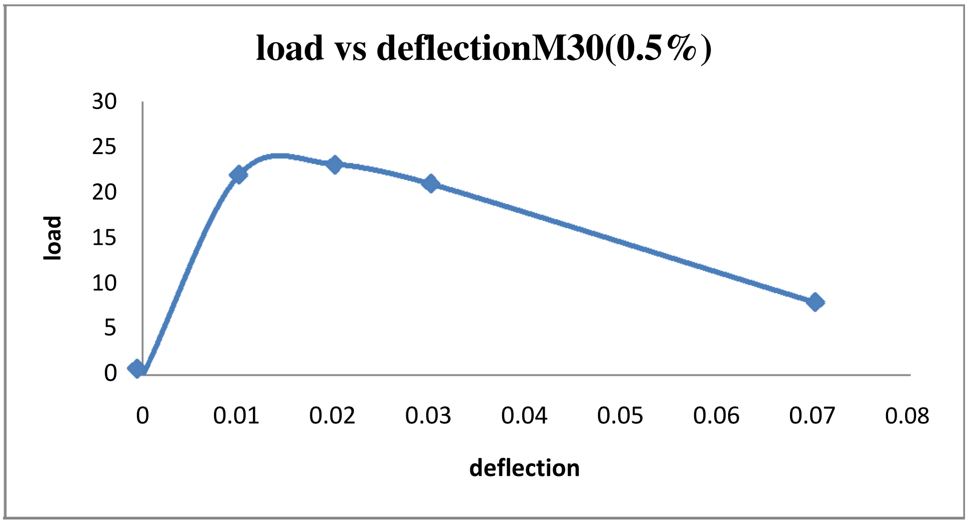
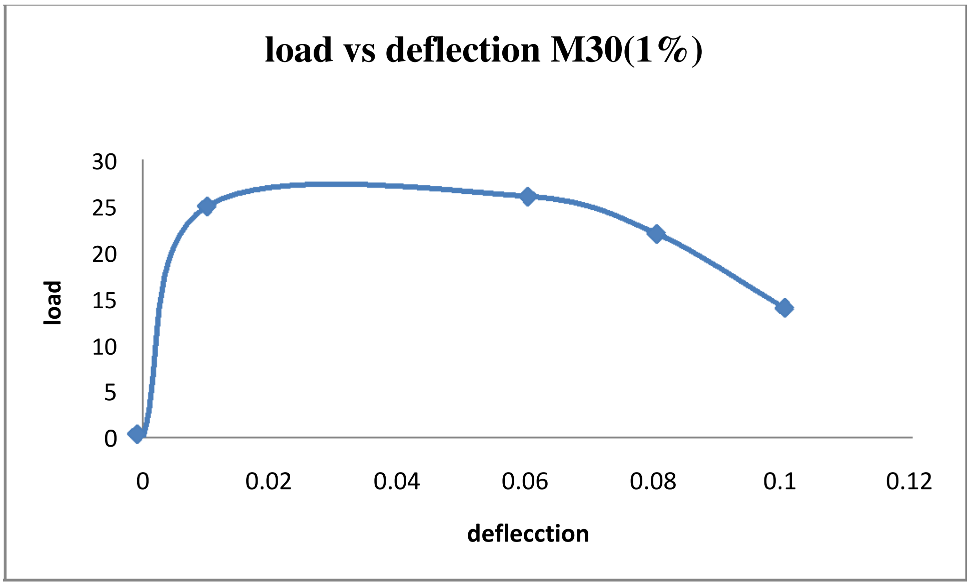
GRADE OF CONCRETE	SPECIMEN TYPE	$F_{CK}(N/MM^2)$	$F_T(N/MM^2)$	DIRECT TENSILE STRENGTH (N/mm^2
M70	0	78.17	5.3	3.24
				2.61
	0.5	80.4	5.8	2.94
				3.6
	1	84.53	6.3	3.52
				3.26
	1.5	91.9	7.1	3.59
				4.70

GRADE OF CONCRETE	SPECIMEN TYPE	$F_{CK}(N/MM^2)$	$F_T(N/MM^2)$	DIRECT TENSILE STRENGTH (N/mm^2
M30	0	44.3	2.3	1.59
				1.36
	0.5	52.04	3.67	3.26
				3.07
	1	56	5.63	3.47
				3.24
	1.5	60.5	6.1	3.69
				3.41




load vs deflectionM30(0.5%)
load
30
25
20
15
10
5
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
deflection


load vs deflection M30(1%)
load
30
25
20
15
10
5
0
0
0.02
0.04
0.06
0.08
0.1
0.12
deflecction

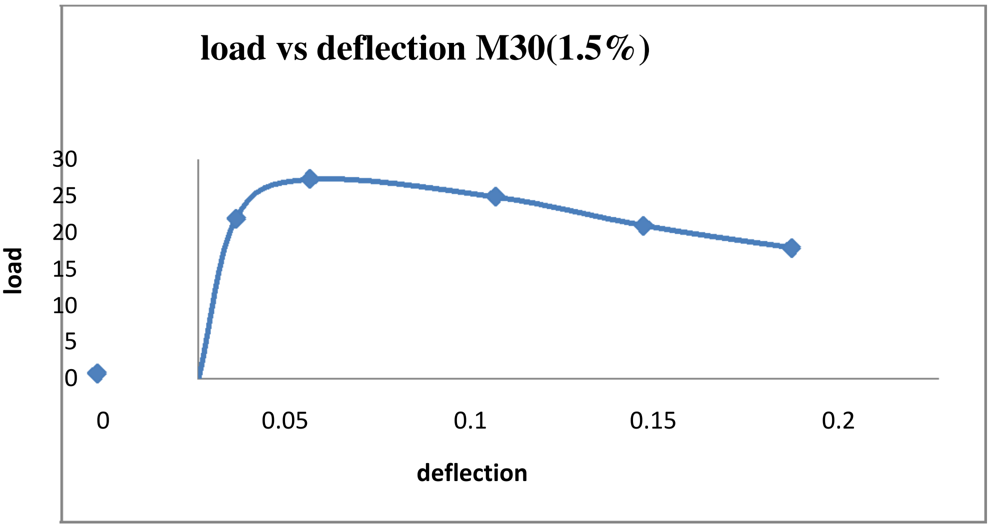
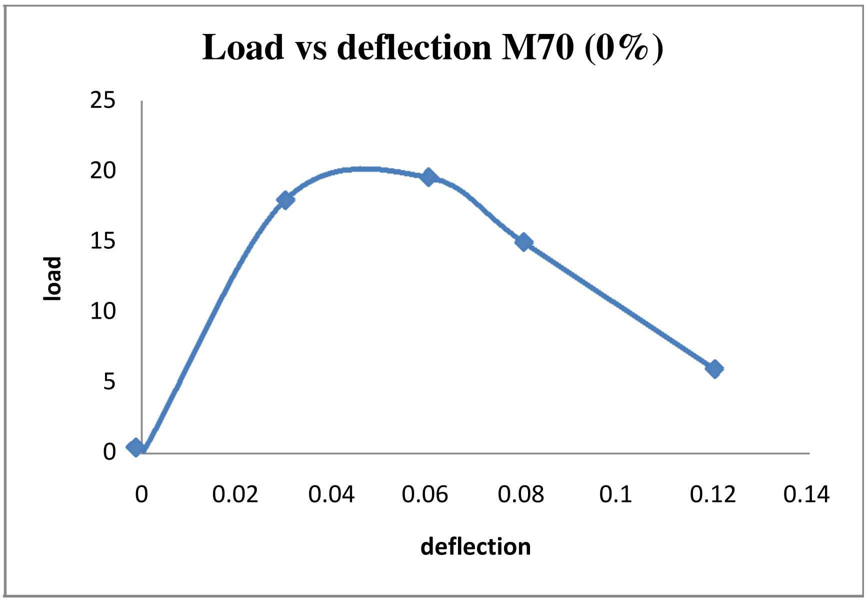
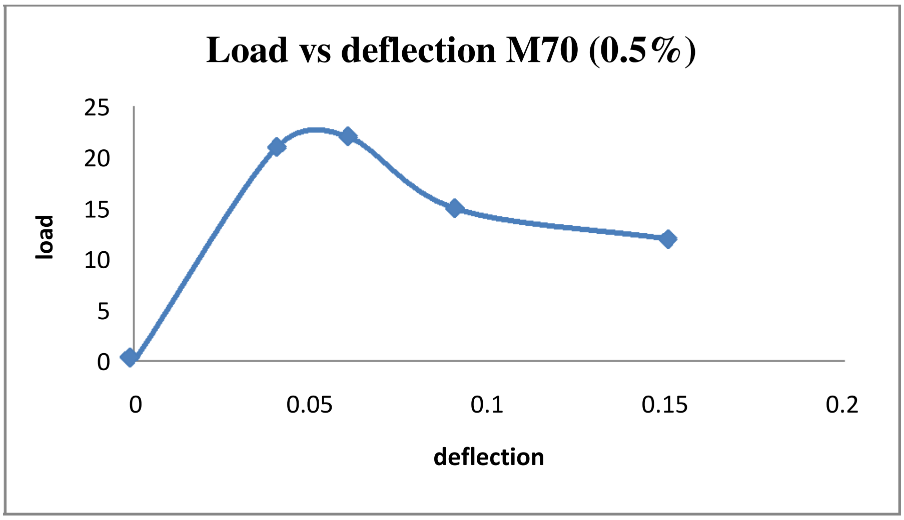
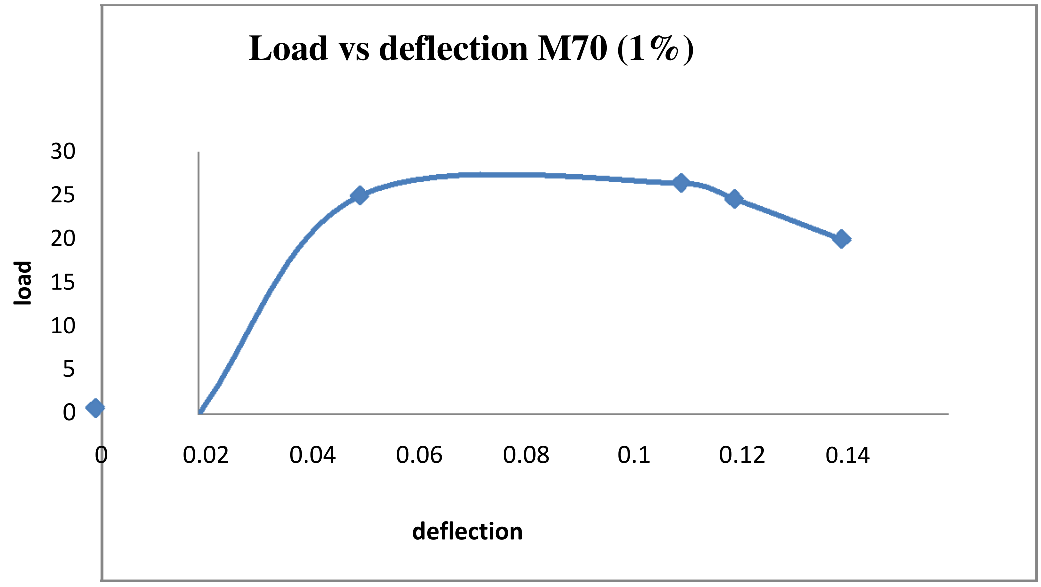


Fig :5.1 load vs deflection for M30(0%,0.5%,1%,1.5%)







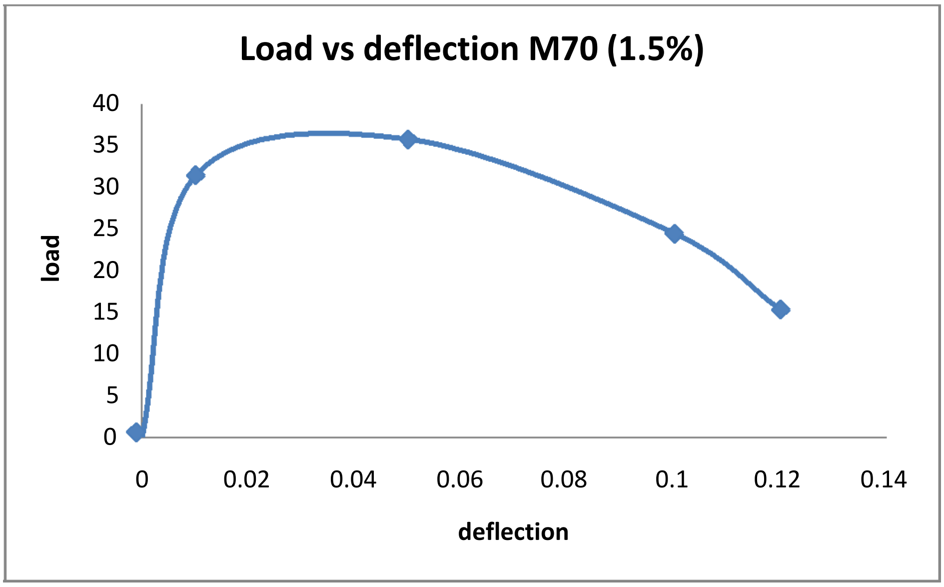
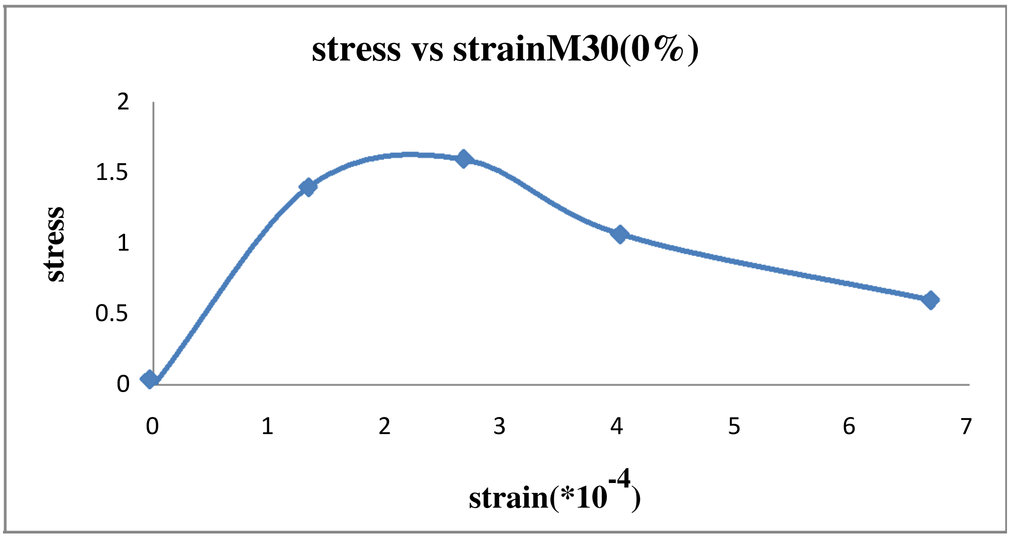
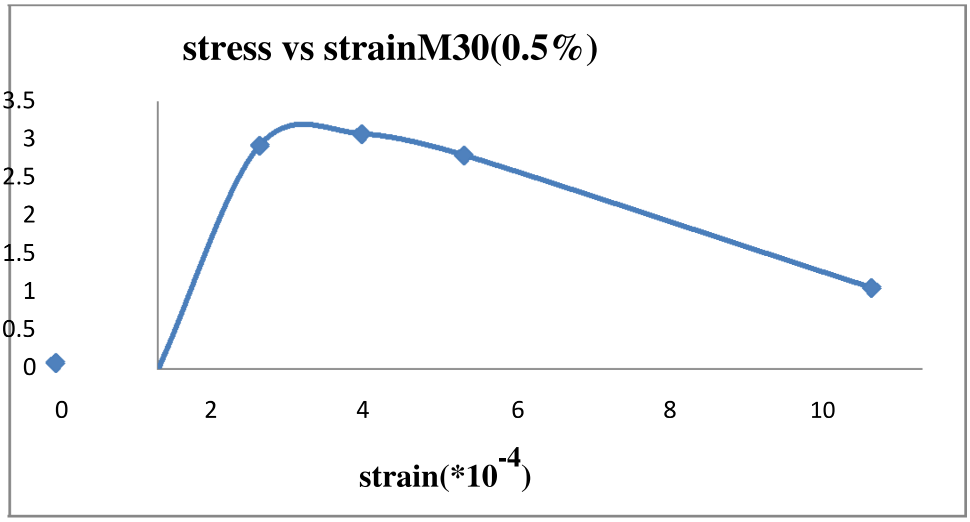
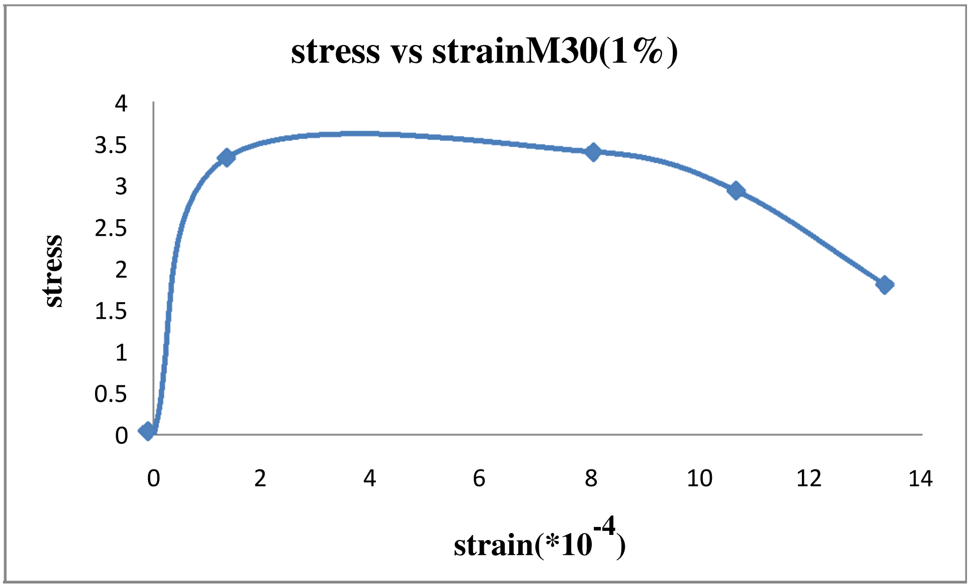


Fig 5.2:load vs deflection M70(0%,0.5%,1%,1.5%)







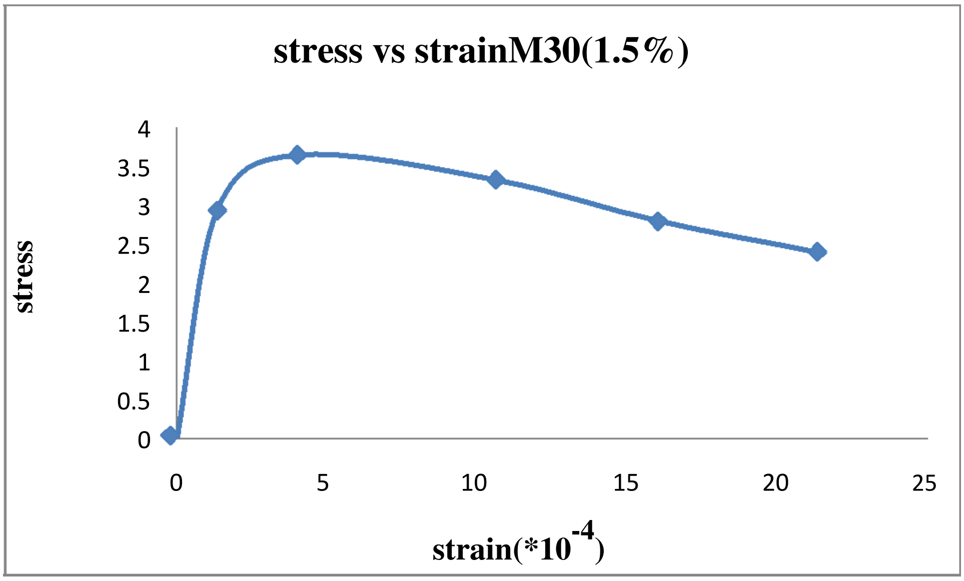
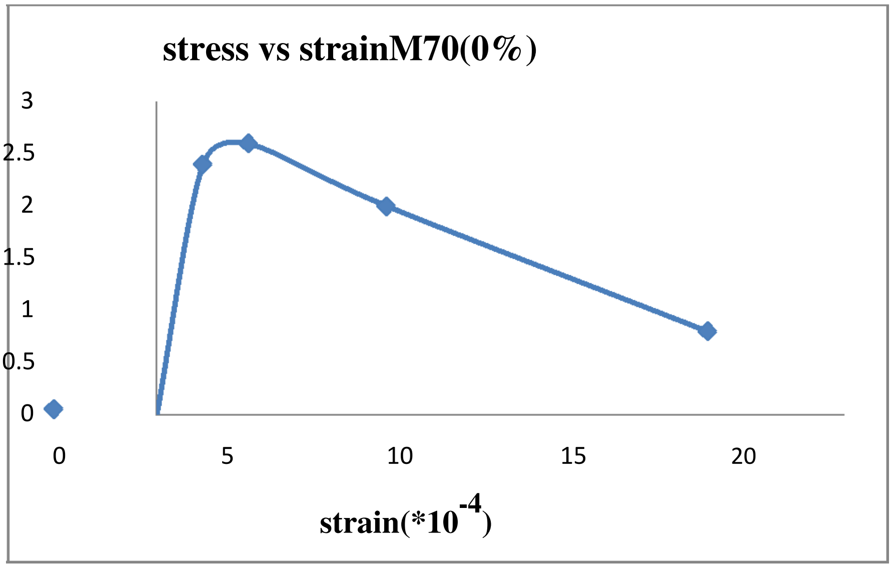
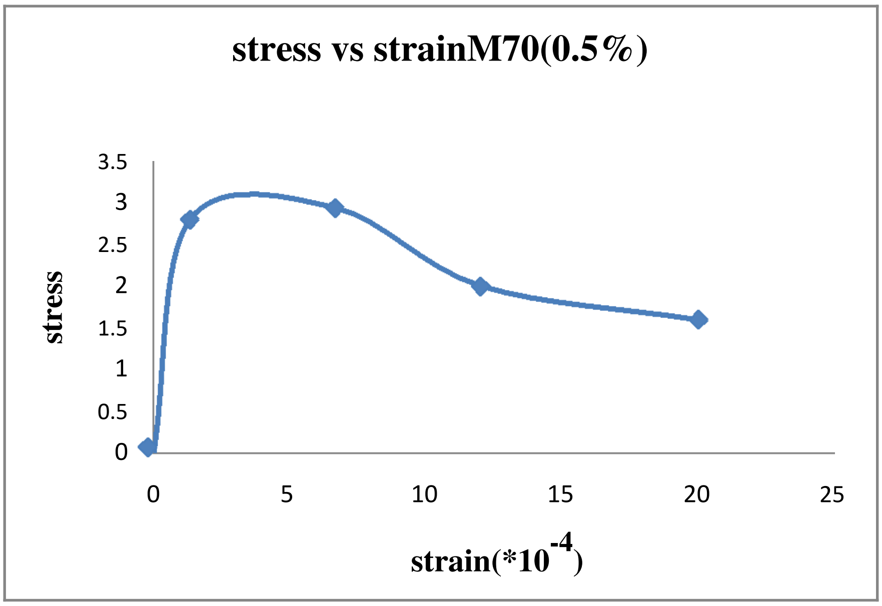
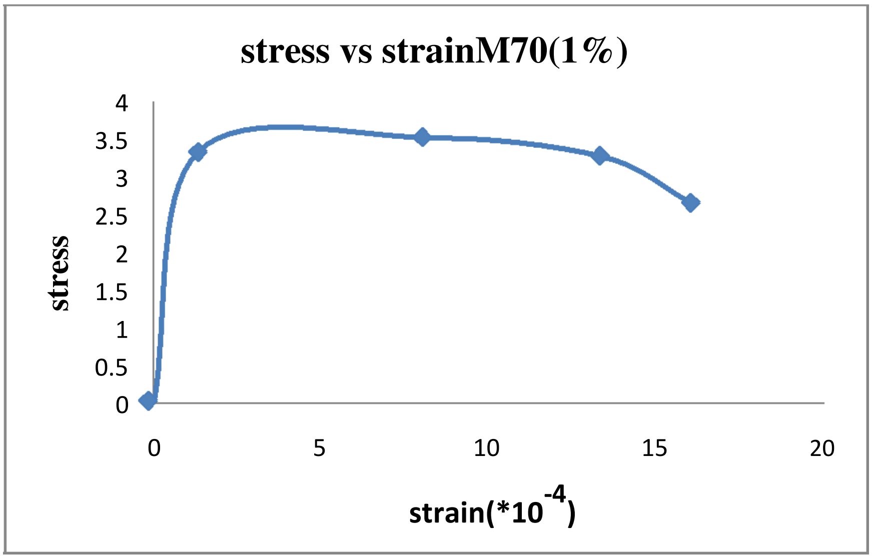


Fig 5.3:stress vs strain ofM30(0%,0.5%,1%,1.5%)


stress vs strainM70(0%)
stress
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
strain(*10^-4)


stress vs strainM70(0.5%)
stress
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25
strain(*10^-4)


stress vs strainM70(1%)
stress
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
strain(*10^-4)

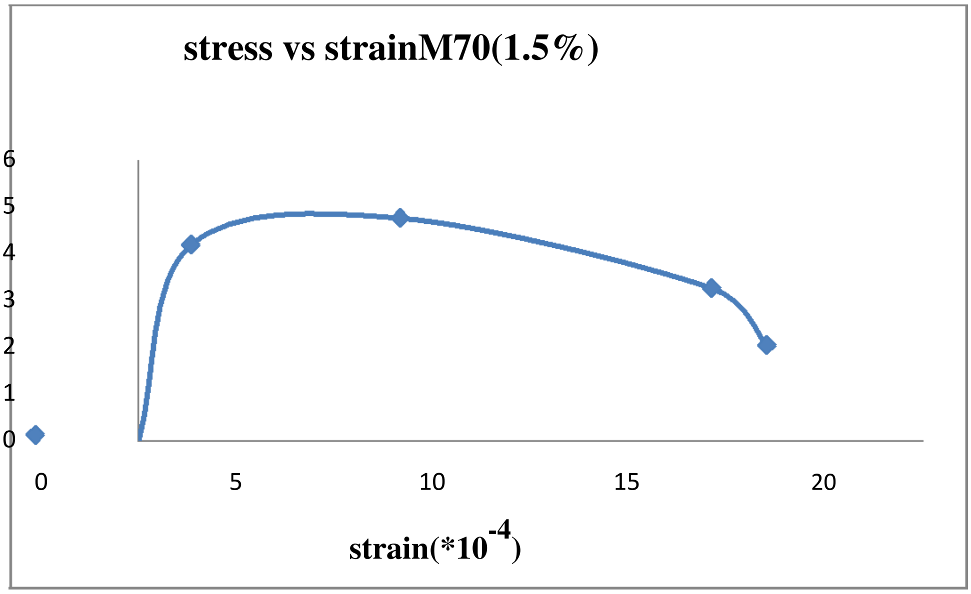
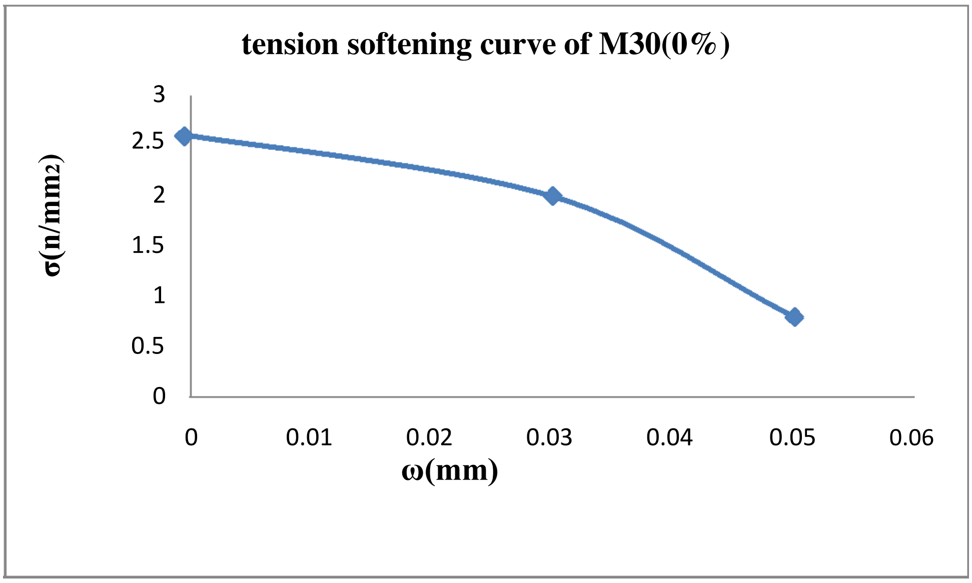
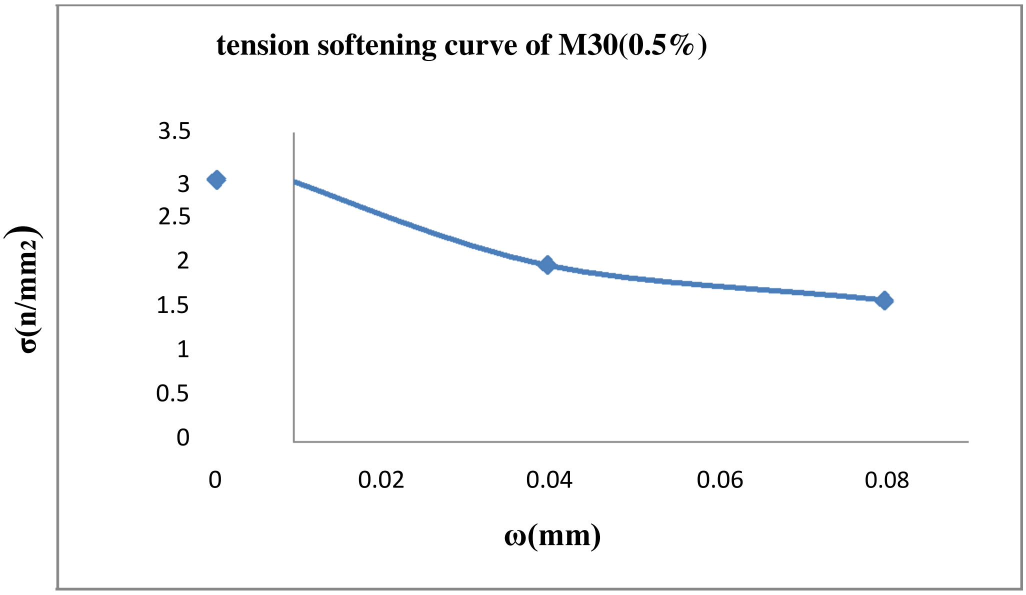
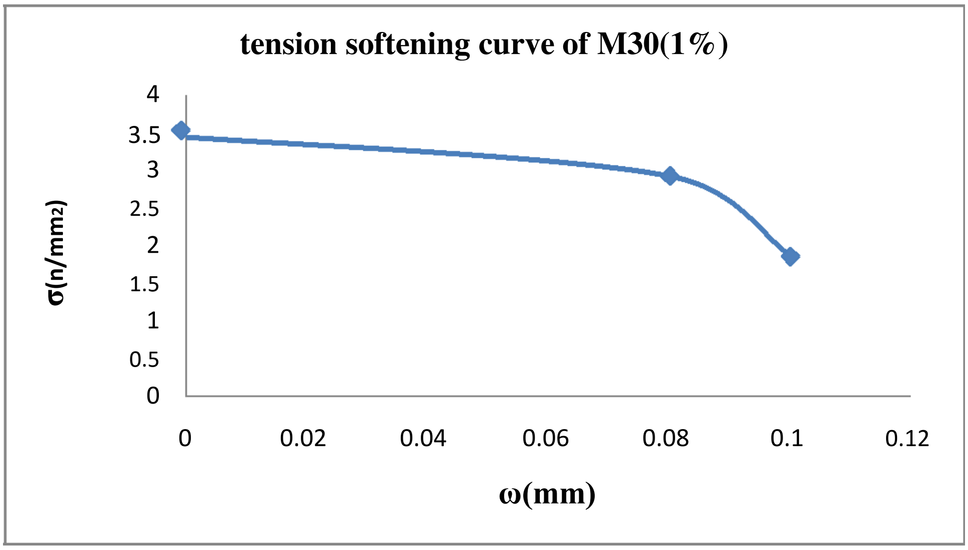


Fig 5.4:strees vs strain ofM70(0%,0.5%,1%,1.5%)







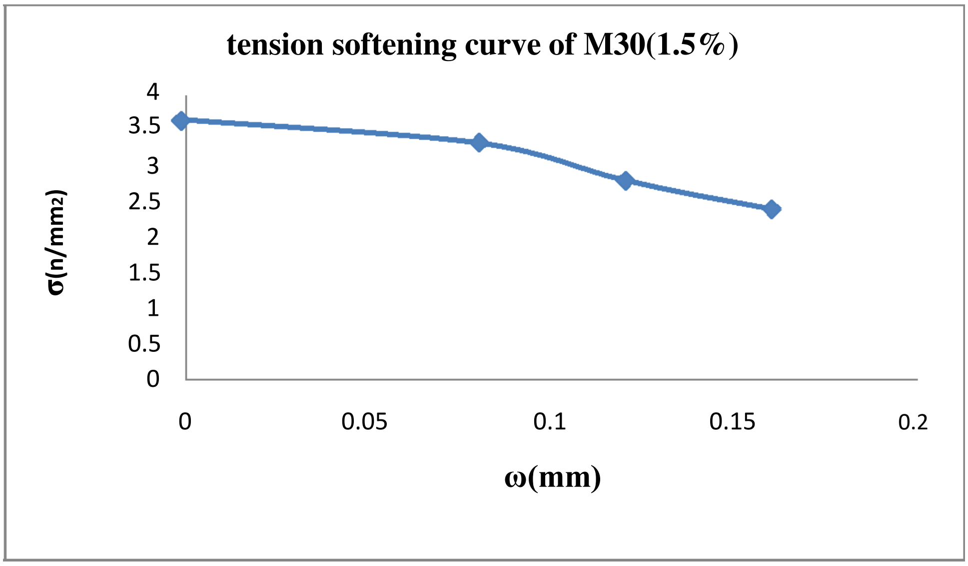
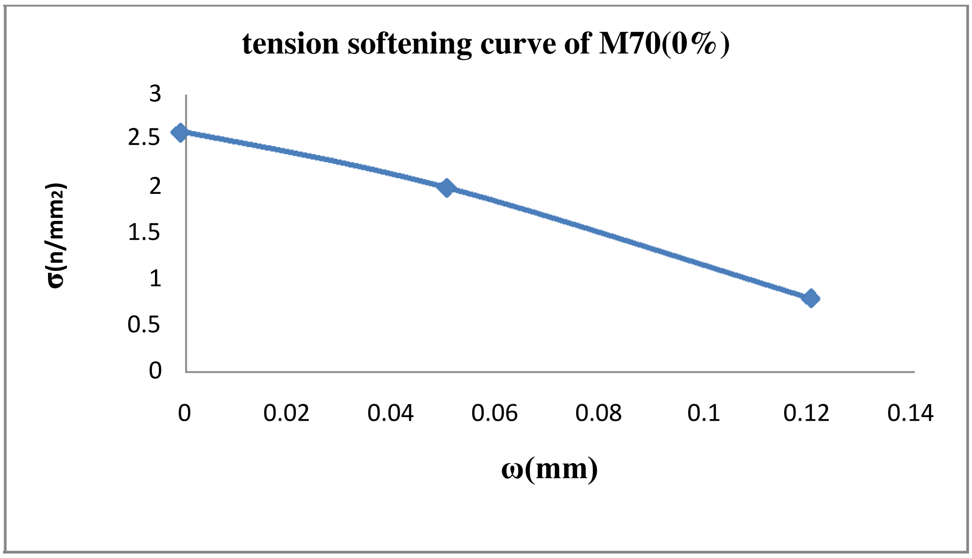
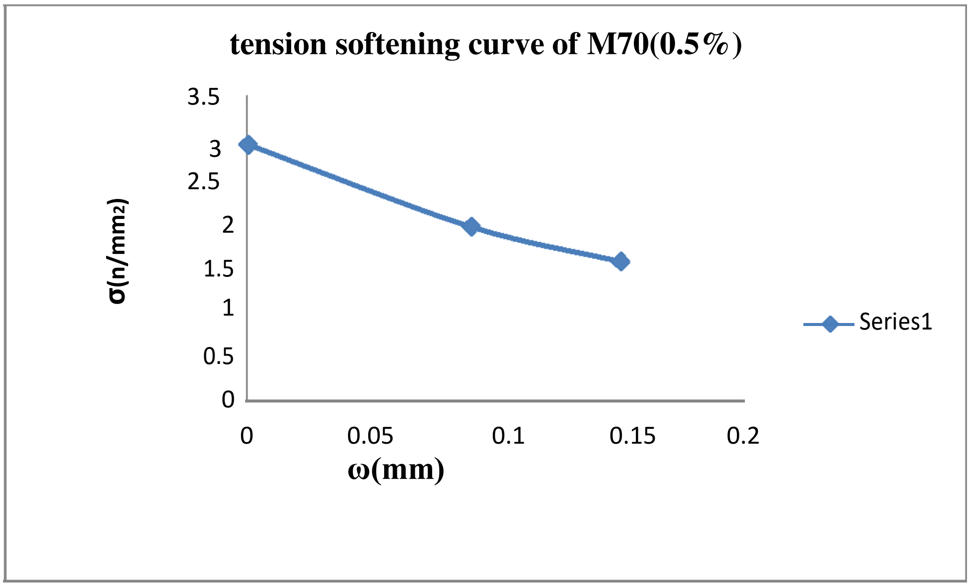
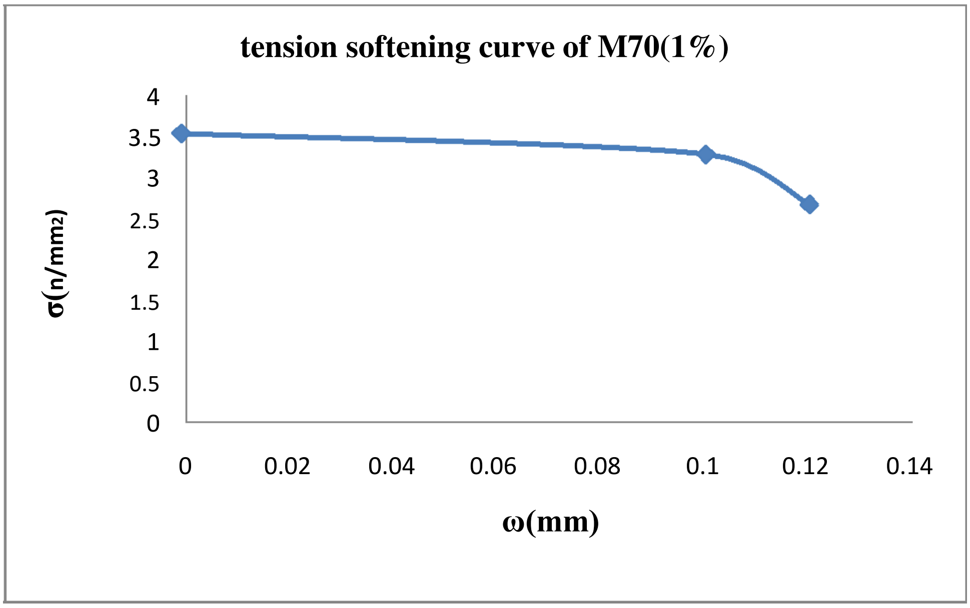


Fig .5.5.Tension softening curves of M30(0%,0.5%,1%,1.5%)


tension softening curve of M70(0%)
σ(n/mm2)
3
2.5
2
1.5
1
0.5
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
ω(mm)


tension softening curve of M70(0.5%)
σ(n/mm2)
3.5
3
2.5
2
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2
ω(mm)
Series1


tension softening curve of M70(1%)
σ(n/mm2)
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
ω(mm)

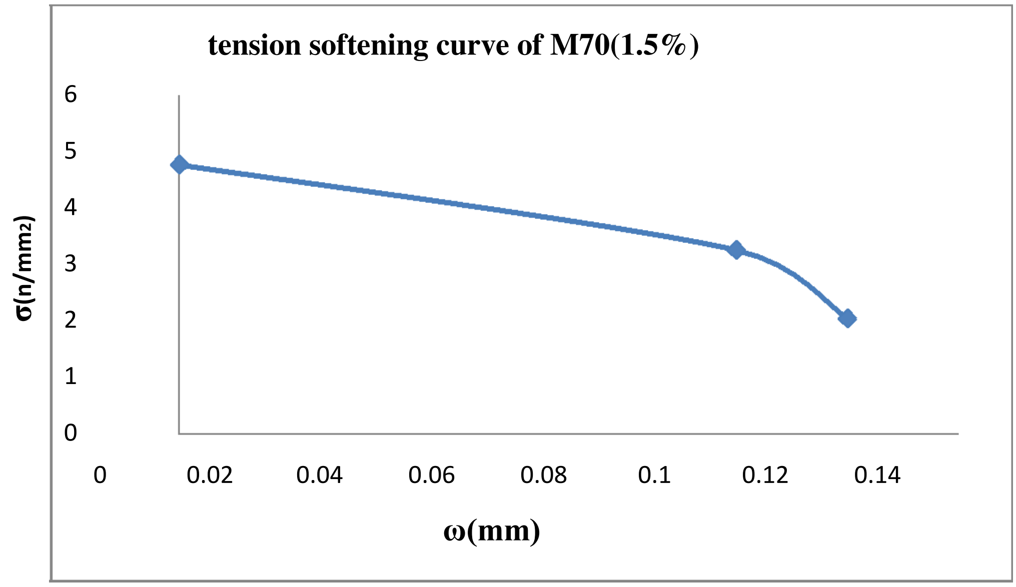


Fig 5.6:tension softening of M70(0%,0.5%,1%,1.5%)

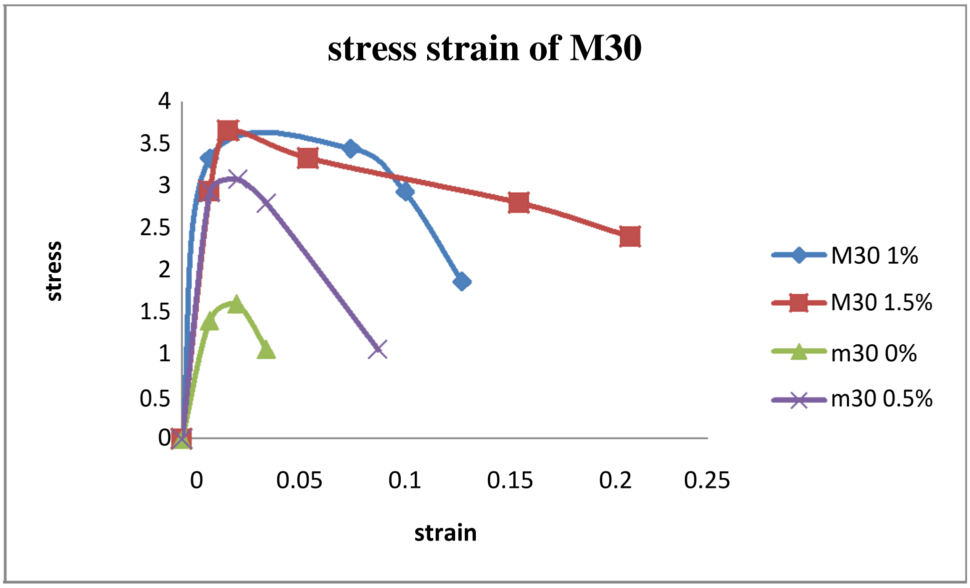


Fig 5.7 : stress strain curves of different % of fibers(M30)

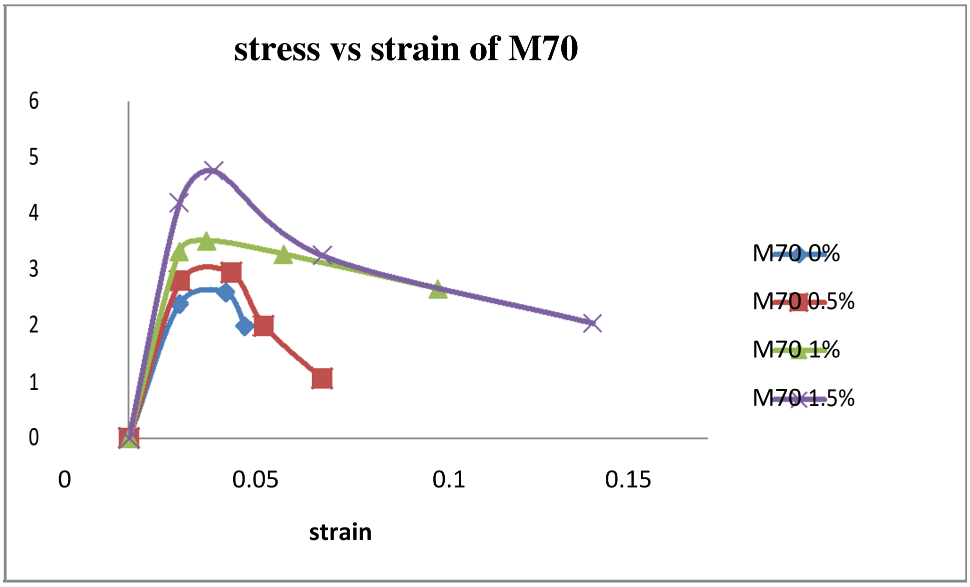


Fig 5.8 : stress strain curves of different % of fibers(M70)

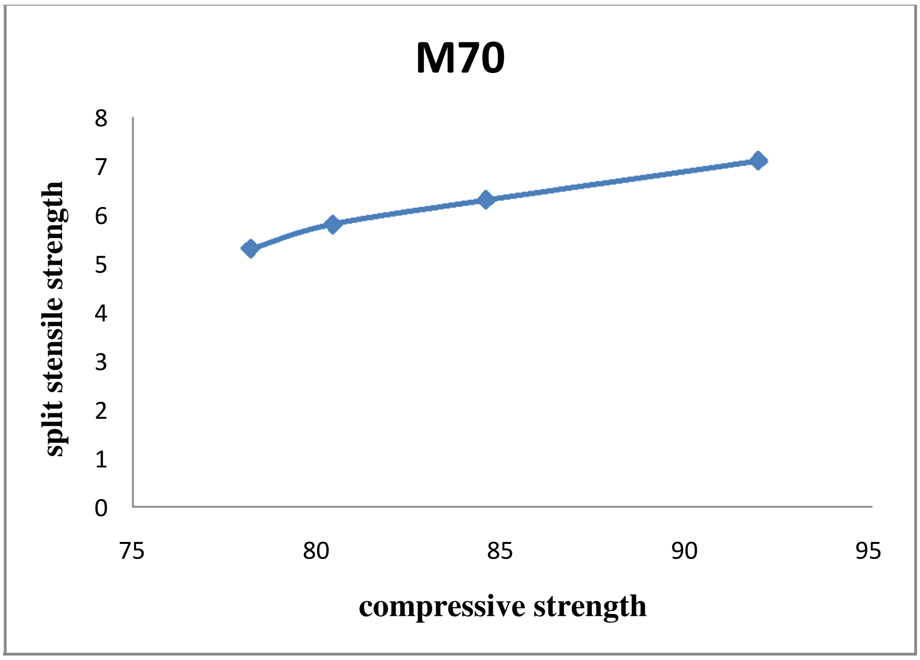


Fig 5.9 relation between split tensile strength and compressive strength for M70

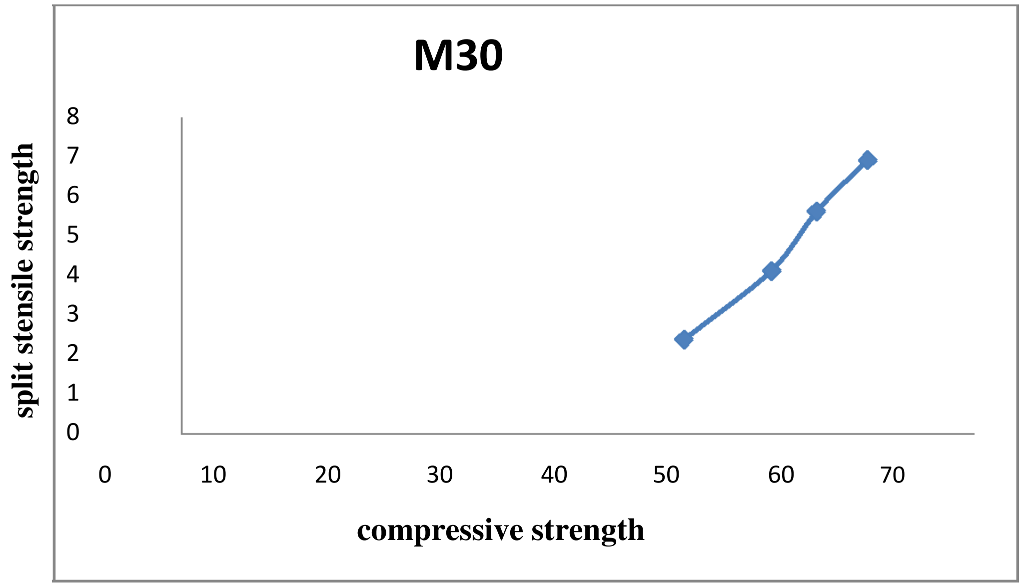


Fig 5.10 relation between split tensile strength and compressive strength for M30

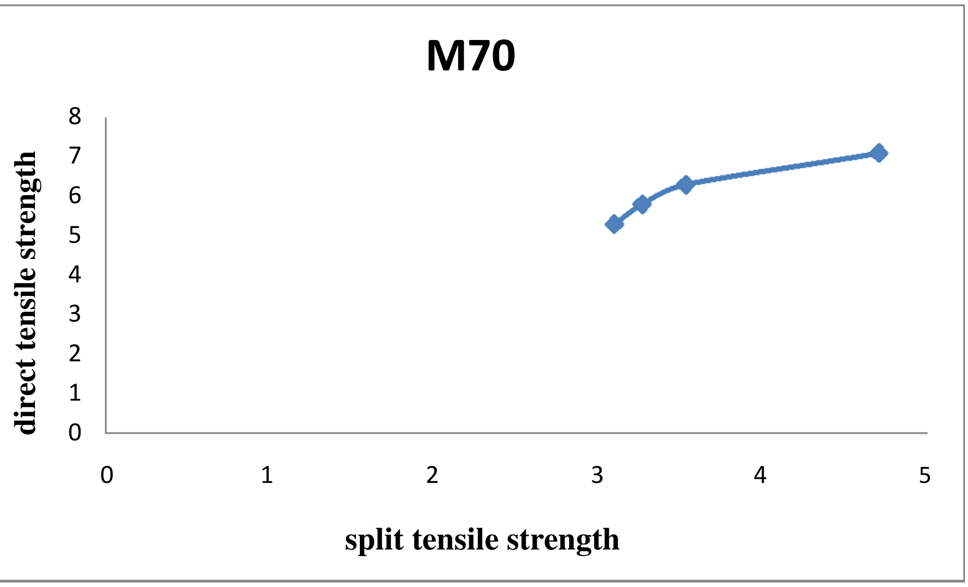


Fig 5.11:Relation between uniaxial –spliting tensile strength M70

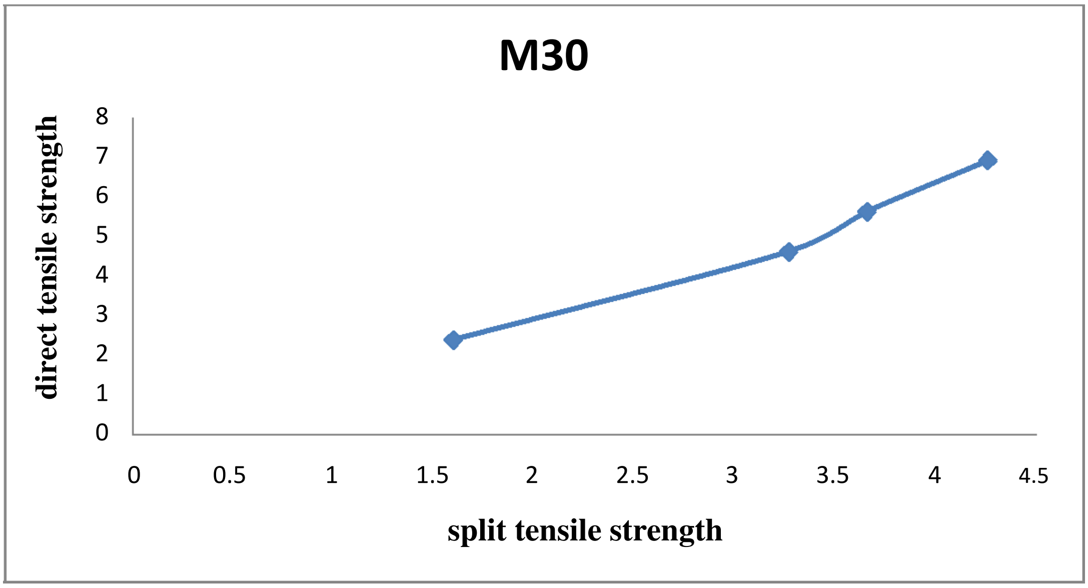


Fig 5.12:Relation between uniaxial –spliting tensile strength M30

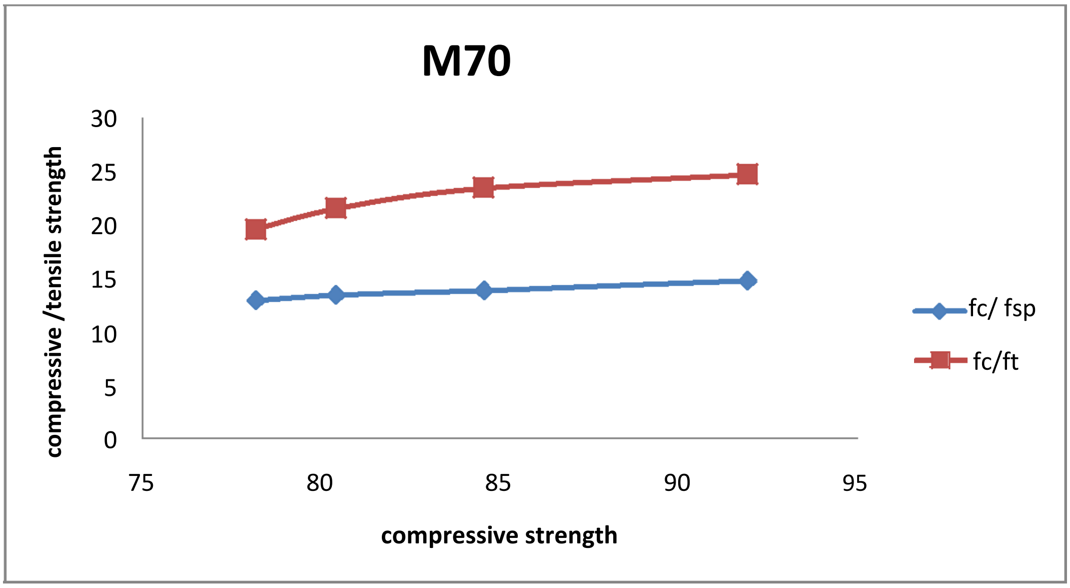


Fig 5.13 variation of compressive /tensile strength ratio with compressive strengthM70

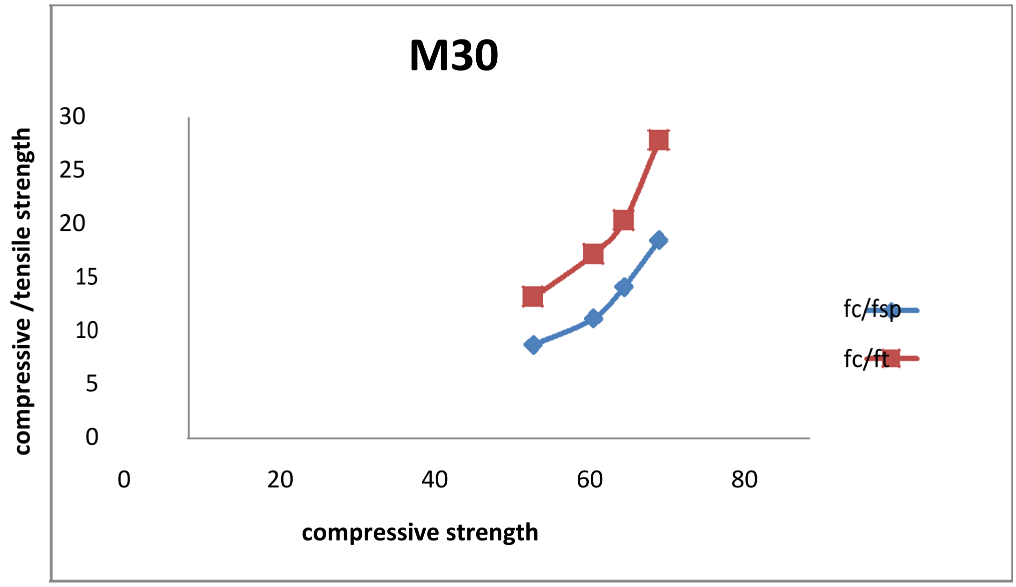


Fig 5.14 variation of compressive /tensile strength ratio with compressive strengthM30

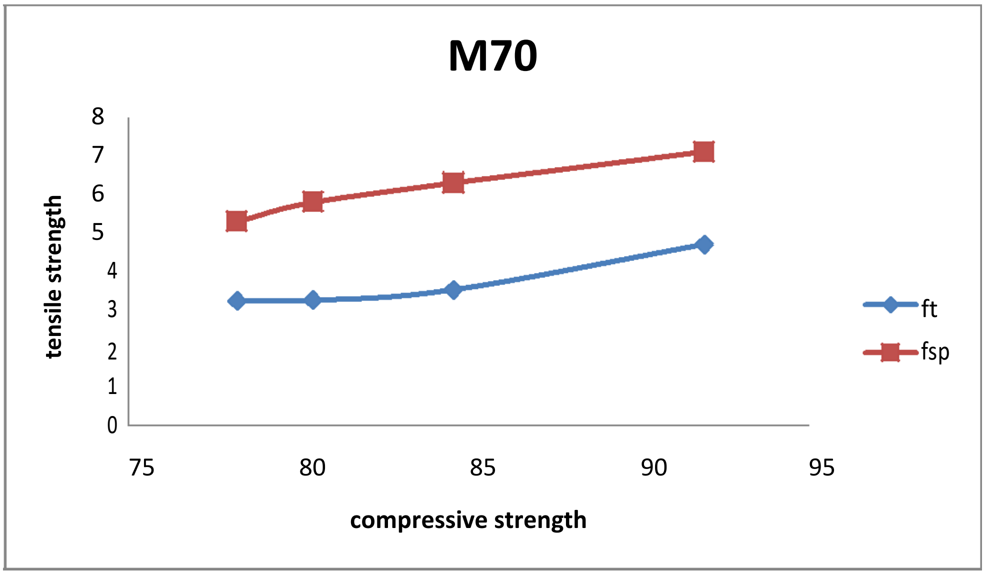


Fig 5.15: relation between compressive strength and tensile strengthsM70

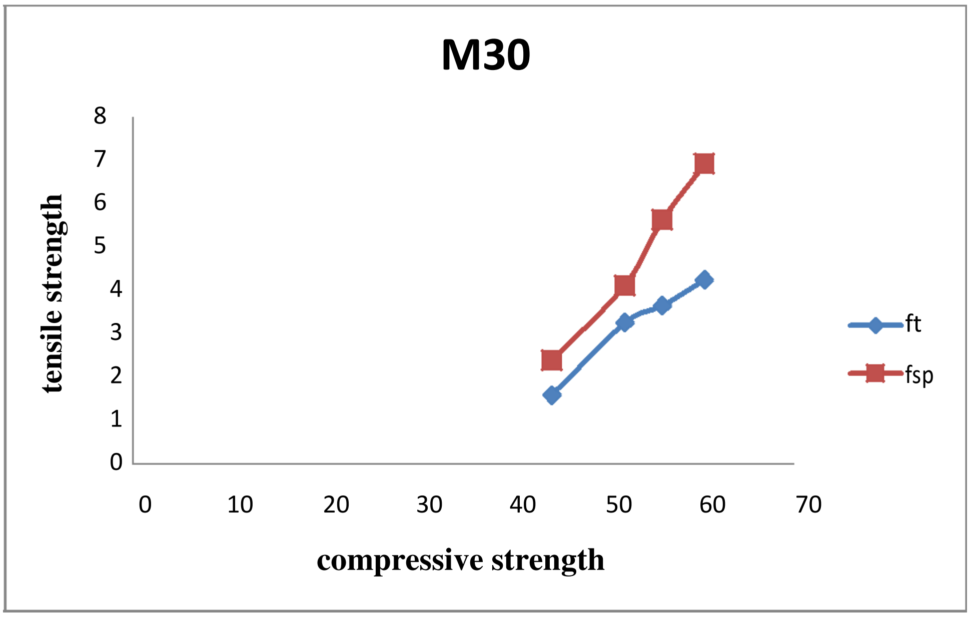


Fig 5.16: relation between compressive strength and tensile strengthsM30

Fig :5.17 failed specimen and frctured surface for M70(0%)

Fig :5.18 failed specimen and frctured surface for M70(0.5%)

Fig :5.19 failed specimen and frctured surface for M70(1%)

Fig :5.20.failed specimen and frctured surface for M70(1.5%)

Fig:5.21 After testing of specimens M70

Fig :5.22 After testing of specimens M30

CHAPTER – 6

CONCLUSIONS

- The proposed test method for measuring direct tensile strength minimized the eccentricity during loading.

- According to the results obtained the uniaxial tensile strength in high strength concrete having 78.17-91.7MPa and 44.5-60.5MPa in normal strength concrete compressive strength.

- It was determined that the uniaxial tensile strength was 61.13% smaller than split tensile strength for 0% replacement and 56.20% smaller for 0.5%replacement and 55.95% smaller for 1% replacement and 66.22% smaller for 1.5% replacement these for high strength concrete .

- It was determined that the uniaxial tensile strength was 66.52% smaller than split tensile strength for 0% replacement and 88.82% smaller for 0.5%replacement and 61.32% smaller for 1% replacement and 60.4% smaller for 1.5% replacement these for normal strength concrete .

- The ratios of split tensile strength to compressive strength and uniaxial tensile strength to compressive strength increased as compressive strength increased.

- Form the results it was proved both in high strength concrete and normal strength the post cracking increase with an increase in % of fiber replacement.

- The higher the volume fraction of fibers, the higher the maximum post-cracking Stress

- The first crack strength and whole post cracking behaviour were mainly influenced by the amount of fibers

- Fracture energy increases with an increase in % of steel fiber both in high strength concrete and normal strength concrete.

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